The Return on Capital in Disaggregated Economies: Theory and Measurement*

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We develop a dynamic general equilibrium model for disaggregated economies with heterogeneous firms and flexible demand and production functions. The model yields a non-parametric, closed-form decomposition of the aggregate return on capital into the risk-free rate and firm-level markups, risk premia, and capital frictions. Applied to U.S. data, we find that, once profits are accounted for, the true return on capital declined from 9% to 5% since 1982, yet remains above the risk-free rate. Capital frictions, particularly among newer cohorts of intangible-intensive firms, are the main barrier preventing convergence. Eliminating them would raise aggregate productivity by 2–13%.

Keywords: Return on capital, risk-free rate, risk premia, misallocation, profits, intangibles.
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1 Introduction

It is well established that the return on risk-free assets in the U.S. has experienced a secular decline since the 1980s, suggesting low returns on savings. However, the return on capital has remained stable over the same period. The divergence between these two returns challenges the standard Neoclassical growth model, which predicts investors should be indifferent across investment opportunities, and has profound aggregate implications.

One interpretation of the secular decline in the risk-free rate is that it signals economic stagnation (Summers, 2014). However, persistently high return on capital challenges this interpretation (Gomme et al., 2015) and forces central banks to choose which rate should guide their policy actions (Reis, 2022). Moreover, persistently high returns on capital, combined with low economic growth, may lead to a growing share of national income accruing to capital owners, potentially fueling wealth inequality (Piketty, 2014).

The existing literature highlights several complementary explanations for the divergence between the return on capital and risk-free assets. Eggertsson et al. (2021) emphasizes the rise in profits, while Caballero et al. (2017) focus on risk premia. Considering both channels, Farhi and Gourio (2018) finds evidence for the importance of each. More recently, Reis (2022) highlights the role of capital frictions. Although all these mechanisms lead to a gap between measured capital returns and the risk-free rate, they have different policy implications: some call for intervention, while others do not. As a result, normative conclusion depends critically on correctly identifying the dominant channel in the data. Yet, no existing study analyzes all these mechanisms jointly, potentially distorting conclusions about their relative importance.

The objective of this paper is to fill this gap. We develop a dynamic general equilibrium theory of the aggregate return on capital in disaggregated economies, allowing for general consumer preferences over goods and flexible producer characteristics, including arbitrary production technologies, markups, risk premia, and capital frictions. In practice, as in Hsieh and Klenow (2009) and Baqaee and Farhi (2020), static firms demand inputs and set prices, generating markups showing up as a distortion to the marginal revenue product of variable inputs, while risk premia and capital frictions enter as wedges that affect the capital-to-variable input ratio. Crucially, these wedges enter the ratio in an isomorphic manner, leading to a non-identification problem.

In contrast to standard approaches in the literature allowing for either risk premia or capital frictions to restore identification, we extend the model by introducing households that lend capital intertemporally to firms in the presence of aggregate risk. This provides an extra pricing condition–a generalized Capital Asset Pricing Model (CAPM)–in which excess returns on capital depend not only on risk premia, but also on markups and capital frictions. By considering the joint behavior of firms and households, we obtain an additional equilibrium condition that restores exact identification and allows us to separate risk premia from capital frictions.

This model yields a new, non-parametric closed-form decomposition of changes in the measured aggregate return on capital—defined as the return on non-variable inputs—into three components: markups, risk premia, and capital frictions. A key distinction is made between the measured and the true return on capital, which coincide only in the absence of profits (Caselli and Feyrer, 2007). This decomposition allows us to characterize the aggregate implications of micro-level changes in markups, risk premia, and frictions through two channels: (i) direct effects, stemming from changes in frictions themselves, and (ii) reallocation effects, arising from shifts in the relative importance of firms driven by those changes.

We demonstrate the empirical relevance and broad applicability of our framework by applying it to conventional firm-level data. Our analysis draws on Compustat data, which offers several advantages: it covers the largest firms, accounting for the majority of aggregate capital (Crouzet and Mehrotra, 2020), and spans a wide range of sectors. The dataset also provides detailed information on sales, costs, and tangible capital, and allows us to measure intangible capital using state-of-the-art approaches from the corporate finance literature (Peters and Taylor, 2017; Ewens et al., 2024). We find that Compustat closely tracks that in the national accounts, making it suitable to study the aggregate return on capital.

The model yields a three-stage identification procedure of markups, risk premia, and capital frictions. First, markups are recovered from the static first-order condition of the variable input, following De Loecker and Warzynski (2012), with the production function identified under imperfect competition without price data as in Ackerberg and De Loecker (2021). Second, risk premia are estimated based on the CAPM derived from the household consumption–saving decision using empirical factors from Hou et al. (2015). Finally, capital frictions are inferred from the capital first-order condition.

To strengthen the credibility of our measures, we apply the methodology of Bils et al.

(2021) to assess how much of the measured capital frictions can be attributed to measurement error, finding that it accounts for 26% on average. We also subject our measures of markups, risk premia, and capital frictions to a range of robustness checks, including alternative production function estimators, different markup measures, and alternative factor models to estimate the CAPM. The results remain largely unchanged across these specifications. With this validation in place, we now turn to the main empirical findings of the paper:

Fact 1: *Since 1982, 20% of the divergence between the return on capital and the risk-free rate is due to profits, while the rest is due to the missing decline in the true return on capital.*

Fact 2: *Contrary to previous findings, the rising frictions associated with capital–rather than the risk premium, which has remained relatively stable over time–have been the main force behind the divergence between the true return on capital and the risk-free rate.*

Fact 3: *Changes in the sectoral composition of the U.S. economy played no role in the rise of frictions associated to capital.*

Fact 4: *The increase in the capital frictions is a within-firm phenomenon associated with newer intangible-intensive cohorts of firms.*

These four facts offer a novel perspective on the divergence between the return on capital and the risk-free rate. A significant portion of this gap is driven by rising profit rates, which have been misattributed to the return on capital. This is consistent with the increase in markups reviewed by Syverson (2024). Once profits and measurement error are properly accounted for, we find that, since 1982, the true return on capital has declined from roughly 9% to 6%. Despite this decline, the estimated return on capital remains above both the average growth rate of GDP per capita and the risk-free rate. This confirms the observation by Piketty (2014) that capital returns have exceeded economic growth—and, at least in theory, wage growth—and reinforces the concern raised by Reis (2022) regarding the dilemma faced by central banks in choosing whether to anchor policy rates to the risk-free rate or to the return on capital.

Our analysis finds that the main factor preventing convergence between the true return on capital and the risk-free rate is the rise in capital frictions, not risk premia which appear stable over the the sample period.[1] This conclusion follows from our empirical methodology, which estimates risk premia in the presence of both markups and capital frictions—unlike prior works attributing excess returns to either risk or frictions only. The rise in capital frictions reflects within-sector changes in firm composition rather than a shift in the sectoral structure of the U.S. economy. Newer cohorts of large firms, which rely more on intangible investment, are found to face significantly higher capital wedges, aligning with evidence that intangible capital is harder to finance and adjust.

Finally, by introducing additional structure on consumer demand and producer technology, we examine the aggregate implications of capital frictions. We find that these frictions generate excess dispersion in the cost of capital across firms, thereby reducing allocative efficiency. Counterfactual experiments suggest that removing these frictions could yield aggregate productivity gains ranging from 2% to 13%.

Despite their generality, our results have limitations. First, the framework abstracts from producers' entry and exit dynamics and international factors related to trade in goods and capital, which may also influence the aggregate return on capital. Second, we model markups, risk premia, and capital frictions as exogenous wedges. The advantage is that we characterize the response of the equilibrium to a change in the wedges without committing to any particular theory of wedge determination. The downside is that this makes it hard to perform counterfactuals when wedges are endogenous. However, in these cases, our results are still relevant as part of a larger analysis that accounts for the endogenous response of wedges.

Related literature. This paper relates to several strands of literature. First, we contribute to the misallocation literature by developing a novel, closed-form disaggregated dynamic general equilibrium model that explicitly links markups, risk premia, and capital frictions to the aggregate return on capital. The seminal work of Hsieh and Klenow (2009) used standard data to quantify the aggregate productivity impact of firm-level frictions captured by wedges. Baqaee and Farhi (2020) generalize their approach to economies with arbitrary input-output linkages and flexible production and demand systems. David and Venkateswaran (2019) separate capital frictions from markups, while Bils et al. (2021) focus on measurement error. Our

[1]This finding of stable risk-premie is in line with standard proxies for risk and estimates from the empirical asset pricing literature reviewed in Section 4.3.2.

contribution is to integrate the household's dynamic consumption-saving decision into this framework, allowing for a modern asset pricing perspective that separates capital frictions from risk premia. In this respect, we are close to David et al. (2022), who develop a firm-level parametric model of risk premia and adjustment costs. In contrast, our approach is non-parametric, accommodates markups and broad capital frictions beyond adjustment costs, and can be implemented directly on standard data.

Second, our paper is related on the extensive literature on CAPM, dating back to Sharpe (1964), Treynor (1962), Lintner (1965a,b), and Mossin (1966). We present a generalization of the standard CAPM in the presence of markups and capital frictions applied to firm-level returns on capital. We show that insights from modern asset pricing are a key addition to standard firm-level production frameworks to perform a comprehensive firm-level anatomy of the aggregate return on capital.

Third, we contribute to the literature on the divergence between the return on capital and the risk-free rate by presenting a model that provides a closed-form decomposition of the joint role of the main firm-level explanations. In contrast, most existing studies focus on one factor at a time, with their conclusions being limited by the ability of aggregate data to distinguish between them. For example, Eggertsson et al. (2021) emphasize rising profits; Caballero et al. (2017) and Marx et al. (2021) focus on risk premia; and Reis (2022) highlights capital frictions. Closest to our approach is Farhi and Gourio (2018), who first stressed the importance of jointly analyzing multiple factors, but exclude capital frictions and conclude that risk premia are the dominant force. Our findings challenge this view by identifying capital frictions—not risk premia—as the key driver. This result stems from our firm-level measurement strategy, which isolates risk premia from confounding effects such as markups and capital frictions.

Finally, showing that the central role of capital frictions is largely driven by intangible-intensive firms, we contribute to the growing literature on the macroeconomic implications of intangible capital for Tobin's Q and investment behavior (Crouzet and Eberly, 2018, 2023). Frictions related to intangibles include adjustment costs (Peters and Taylor, 2017; Belo et al., 2022; Chiavari and Goraya, 2024) and financial constraints (Caggese and Pérez-Orive, 2022; Falato et al., 2022). Moreover, the fact that capital frictions are concentrated among newer cohorts of large, intangible-intensive firms contributes to the literature emphasizing the important of ex-ante firm heterogeneity (Sterk et al., 2021; Moreira, 2016; Sedláček and Sterk,

2017) and cohorts effects (Bowen III et al., 2023; Ma et al., 2024).

Outline. Section 2 reviews the facts on the divergence between the return on capital and the risk-free rate, along with key explanations from the literature. Section 3 introduces our framework. Section 4 describes the data and measurement. Section 5 presents the main results, and Section 6 discusses macro implications and 7 concludes.

2 Return on Capital and Risk-Free Rate Divergence

This section reviews the basic facts about the evolution of the return on capital and the risk-free rate since the 1980s, the period most often studied in the literature (e.g., Farhi and Gourio, 2018). While the risk-free rate is typically observed directly in the data, the return on capital is a model-implied concept, which we discuss in detail below.

We measure the aggregate return on capital in line with established literature. As noted by Caselli and Feyrer (2007), under the standard assumptions of constant returns to scale and perfect competition, the aggregate net output can be expressed as follows:

$$Y_t - \delta K_t = (R_t - \delta) K_t + W_t L_t, \tag{1}$$

where Y represents value added, δ is the depreciation rate, K is total capital, R is the rental price of capital, and WL is total labor. Equation (1) indicates that net output, $Y - \delta K$, is allocated either to payments to labor services, WL, or capital services, $(R - \delta) K$. Hence, while W denotes the return on labor, $R^k \equiv R - \delta$ represents the return on capital, which can be measured as follows:[2]

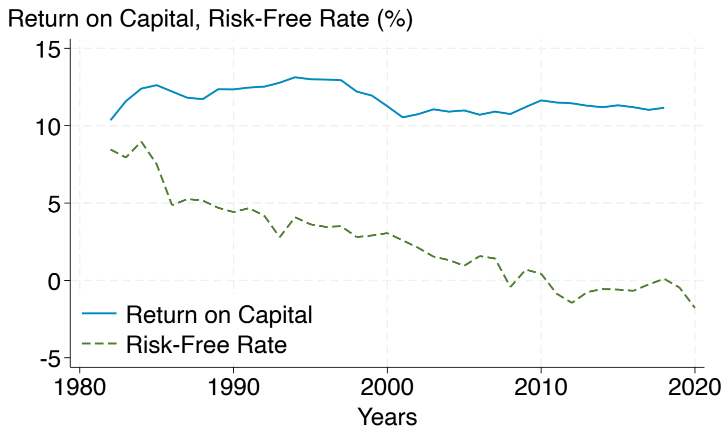
$$R_t^k = \frac{Y_t - \delta K_t - W_t L_t}{K_t}. \tag{2}$$

Note that, since investors should be indifferent between investing one unit of output in capital or an investment yielding a risk-free return r, we would expect R^k to equal r. Figure 1 illustrates the evolution of the return on capital and the risk-free rate. The return on capital is calculated using equation (2) with data from the BEA, while the risk-free rate is represented by

[2]More generally, this methodology for calculating the aggregate return on capital can be applied in the presence of any general set of variable inputs, using the formula $R_t^k = (GO_t - \delta K_t - X_t)/K_t$, where GO is gross output and X denotes total nominal variable input costs, including labor and all types of intermediate inputs.

the real market yield on U.S. Treasury securities with a 10-year constant maturity. Additional details on these calculations are provided in Appendix A.

Figure 1: Evolution of Return on Capital and Risk-Free Rate



Note. Figure 1 shows the evolution of the return on capital and of the risk-free rate, measured in percent, since 1982. The return on capital is measured using equation (2) with NIPA data. The risk-free rate is the market yield on U.S. Treasury securities with a 10-year constant maturity net of expected inflation from Michigan.

Since the 1980s, the risk-free rate has steadily declined, falling from just below 10 percent to near 0 percent. In contrast, the return on capital has remained relatively stable, consistently hovering just above 10 percent. This divergence, following decades of convergence between the two rates (see Appendix A for the historical evolution of these measures), has puzzled researchers and spurred various potential explanations, including markups, risk premia, and capital frictions.

Appendix A demonstrates that alternative measures of the risk-free rate, as outlined in Rachel and Summers (2019), including the real Aaa corporate bond yield, the real Baa corporate bond yield, and the real S&P 500 earnings yield, exhibit a similar downward trend. Further, Appendix A shows that alternative measures of the return on capital based on Gomme et al. (2011) yield similar patterns, with no evidence of a decline over time. They report measures based solely on the business sector, excluding the impact of housing, which produces results closely aligned, with ours which are going to be based on Compustat. Additionally, they present measures both with and without capital gains and before and after taxes, demonstrating that neither adjustment accounts for the missing decline in the return on capital.

3 Model

This section introduces a disaggregated dynamic general equilibrium framework in the presence of heterogeneous markups, risk premia, and capital frictions. It provides a closed-form decomposition of the aggregate return on capital, and characterizes how this is affected by shocks to firm-level risk premia, markups, and capital frictions.

3.1 The Firm Problem

Time is discrete and indexed by t. Firms are static, indexed by $i \in \mathcal{I}$. Firms produce using the following production function:

$$q_{it} = q(z_{it}, \{\ell_{it}\}, \{k_{it}\}), \tag{3}$$

where output quantity is q_{it}, Hicks-neutral productivity is z_{it}, the vector of variable inputs is $\{\ell_{it}\} = (\ell_{it}^1, \ldots, \ell_{it}^N) \in \mathbb{R}^N$, and the vector of capital inputs is $\{k_{it}\} = (k_{it}^1, \ldots, k_{it}^O) \in \mathbb{R}^O$. We define the output-elasticities of variable inputs as $\{\mathcal{E}_\ell\}$ and output-elasticities of capital input as $\{\mathcal{E}_k\}$.

Throughout the remainder of the paper, we will use bold notation for the firm-level variables that we consider as potential drivers of the aggregate divergence documented in Section 2. These include (i) firm-level risk premia, (ii) capital-specific firm-level mismeasurement or frictions, (iii) firm-level markups, and (iv) firm-level fixed costs.

Firms borrow all types of capital from investors, paying a rental rate comprising a risk-free rate r, a firm-level risk premium $\boldsymbol{\zeta}^k$, and a depreciation rate δ^k. Additionally, we account for the possibility of capital-specific frictions, which we represent in reduced form as a wedge $\boldsymbol{\tau}^k$. Appendix B.1 provides various alternative microfoundations for this term, relating it explicitly to either adjustment costs or financial frictions. Finally, firms face an overhead cost $\boldsymbol{f}$ to operate their production technology. The objective function associated with the *firm's cost minimization problem* is:

$$\mathcal{L}(\{\ell_{it}\}, \{k_{it}\}, \xi_{it}) = \sum_{\ell^n} p_t^{\ell^n} \ell_{it}^n + \sum_{k^o} \left(r_t + \boldsymbol{\zeta}_{it}^{k^o} + \delta^{k^o} + \boldsymbol{\tau}_{it}^{k^o}\right) p_t^{k^o} k_{it}^o + \boldsymbol{f}_{it} - \xi_{it} \left(q(\cdot) - q_{it}\right), \tag{4}$$

where p^{ℓ^n} is the price of the variable input $\ell^n \in \{\ell\}$, p^{k^o} is the price of capital input $k^o \in \{k\}$,

ξ is the Lagrange multiplier, $q(\cdot)$ represents the technology as specified in equation (3), and q_{it} is a is a scalar.

We assume that firms that variable and capital input prices as given and that firms set the output price according to $p_{it} = \boldsymbol{\mu}_{it}\xi_{it}$, where $\boldsymbol{\mu}$ is the price-cost markup and ξ_{it} is the marginal cost of production. This is consistent with the general demand structures as specified in the household problem in Section 3.2.

Proposition 1 (Firms' First-Order Conditions) *The firm-level equilibrium conditions are:*

$$\boldsymbol{\mu}_{it} = \mathcal{E}_{\ell^n} \frac{p_{it} q_{it}}{p_t^{\ell^n} \ell_{it}^n} \quad \forall \ell^n, \tag{5}$$

$$r_t + \boldsymbol{\zeta}_{it}^{k^o} + \delta^{k^o} = \mathcal{E}_{k^o} \frac{1}{\boldsymbol{\mu}_{it}} \frac{p_{it} q_{it}}{p_t^{k^o} k_{it}^o} - \boldsymbol{\tau}_{it}^{k^o} \quad \forall k^o. \tag{6}$$

Proof. See Appendix B.2. ■

Note that equation (5) determines the markups as a function of observables, akin to Hall (1988) and De Loecker and Warzynski (2012). Our framework nests the structure of Hsieh and Klenow (2009) as a special case, where equations (6) determine the revenue-based marginal products of capital ($MRPK^o$), as defined by the right-hand side of each equation. However, unlike their approach, where the capital first-order conditions allowed for an unobservable in the wedge $\boldsymbol{\tau}^{k^o}$, we also consider that heterogeneity in firm-level risk premia $\boldsymbol{\zeta}^{k^o}$ may contribute to the variation in the revenue-based marginal product of capital.

Equations (5) and (6) show an important non-identification result. Looking only at the firm side of the problem, without fully specifying the asset supply coming from the household side of the economy, it is not possible to separately identify the markups, capital frictions, and risk premia. This is why, after defining the difference between true and observable revenue-based marginal product of capital, we devote our attention to the household decisions and the insights we can derive from them for the determination of risk premia. We define the observable revenue-based marginal product, $\widetilde{MRPK^o}$, as follows:

$$\widetilde{MRPK}^o_{it} = MRPK^o_{it} + \boldsymbol{\tau}^{k^o}_{it}, \quad \forall k^o. \tag{7}$$

This implicitly states that the true revenue-based marginal product of capital is equal to the risk-free rate r, the risk premium $\boldsymbol{\zeta}^{k^o}$, and the depreciation rate δ^{k^o}.

3.2 Household Side

The economy is populated by a representative household that makes consumption-saving decisions. The representative maximizes the expected utility,

$$\max_{C_t, B_{t+1}, \{c_{it}, \{k^o_{it+1}\}_{\forall k^o}\}_{\forall i}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t; Z_t), \quad \text{and} \quad C_t \equiv \mathcal{D}(\{c_{it}\}); \tag{8}$$

subject to the following budget constraint:

$$P_t C_t + B_{t+1}$$
$$\leq \sum_{i \in \mathcal{I}} \sum_{k^o \in \{k\}} \left(\underbrace{\left(r_t + \boldsymbol{\zeta}^{k^o}_{it} + \delta^{k^o}\right)}_{MRPK^o_{it}} p^{k^o}_t k^o_{it} - p^{k^o}_{t+1} \left(k^o_{it+1} - (1 - \delta^{k^o}) k^o_t\right) \right) + (1 + r_t) B_t + \Pi_t; \tag{9}$$

where β is the discount factor, C is aggregate consumption, Z is a discount factor shock, $U(\cdot)$ is utility over aggregate consumption, $\mathcal{D}(\cdot)$ is the demand aggregator over the heterogeneous good, B is the risk-free bond, and Π are aggregate profits.

Our specification allows for arbitrary functional forms of the demand aggregator and the only assumptions needed are that the individual demand curves are downward sloping in prices and there exists an ideal price index P. An implicit assumption in equation (9)—dating back at least to Mossin (1966) and standard in neoclassical models—is that the return on household savings equals the return on firms' investments. Moreover, since we consider a closed economy, equation (9) naturally embeds the assumption that households own all firms and receive their profits. However, this ownership structure can be generalized to a more realistic equity market, which would introduce additional asset pricing equations and yield a well-defined notion of firm-level equity premia. Although such an extension is feasible in our framework, we abstract from it for simplicity, as firm-level equity premia would not affect the results presented below on the return on capital and would introduce unnecessary complexity.

Proposition 2 (Generalized CAPM) *The asset-pricing conditions can be summarized by,*

$$1 = \mathbb{E}_{t-1}\left[M_t \left(MRPK_{it}^o + (1-\delta^{k^o})\frac{p_{t+1}^{k^o}}{p_t^{k^o}}\right)\right] \quad \forall i \in \mathcal{I}, \quad \forall k^o \in \{k\}, \tag{10}$$

where M_t represents the stochastic discount factor. These conditions lead to the following generalized CAPM:

$$\widetilde{MRPK}_{it}^o - r_{t-1} - \delta^{k^o} = \boldsymbol{\tau}_{it}^{k^o} - g_{it}^{k^o} + \overbrace{\underbrace{\frac{\mathbb{C}_{t-1}\left[M_t, \widetilde{MRPK}_{it}^o\right]}{\mathbb{V}_{t-1}\left[M_t\right]}}_{\equiv \boldsymbol{\beta}_{it}} \underbrace{\left(-\frac{\mathbb{V}_{t-1}\left[M_t\right]}{\mathbb{E}_{t-1}\left[M_t\right]}\right)}_{\equiv \boldsymbol{\lambda}_t}}^{\equiv \boldsymbol{\zeta}_{it}} + \varepsilon_{it}^o, \quad \forall i \in \mathcal{I}, \quad \forall k^o \in \{k\}, \tag{11}$$

where $g^{k^o} \equiv (1-\delta^{k^o})\left(\frac{p_{+1}^{k^o}}{p^{k^o}} - 1\right)$. Thus, the firm-level excess return can be expressed as a function of the wedge $\boldsymbol{\tau}_{it}^{k^o}$, capital gains $g_{it}^{k^o}$, a risk premium $\boldsymbol{\zeta}_{it}^{k^o} \equiv \boldsymbol{\beta}_{it}^{k^o} \boldsymbol{\lambda^{k^o}}_t$, and an expectational error ε_{it}^o.

Proof. See Appendix B.3. ■

Equation (11) expresses the risk premium as the product of two components: the exposure of the return to movements in the stochastic discount factor—i.e., the firm's riskiness—summarized by $\boldsymbol{\beta}_{it}$, and the aggregate price of risk, summarized by $\boldsymbol{\lambda}_t$. Thus, it generalizes standard capital asset pricing models by explicitly accounting for wedges $\boldsymbol{\tau}_{it}$ and the presence of markups $\boldsymbol{\mu}_{it}$ through the observed revenue-based marginal product of capital $\widetilde{MRPK}_{it}$.[3]

Similarly to equations (5)-(6), equation (11) also highlights the importance of considering all narratives together. Ignoring the role of markups and capital-specific wedges would lead to

[3] It is important to note that ignoring the presence of markups and using the usual measure of the revenue-based marginal product of capital unadjusted for firm-level markups, $\mathcal{E}_{k^o}(p_{it}q_{it})/(p_t^{k^o}k_{it}^o)$, as dependent variable would lead to a bias in the regression given by the following $\widetilde{MRPK}_{it}^o(\boldsymbol{\mu}_{it}-1)/\boldsymbol{\mu}_{it}$, which vanishes only in the presence of perfect competition.

incorrectly attributing changes in the gap between observed revenue-based marginal product and the risk-free rate to shifts in the risk premium, thus overemphasizing its role in this divergence.

3.3 Model-Driven Identification

Equations (5), (6) and (11) define a system of equations with several unknowns: markups $\{\boldsymbol{\mu}\}$, risk premia $\{\boldsymbol{\zeta}\}$, and capital-specific wedges $\{\boldsymbol{\tau}\}$. In particular, for each firm, we have $N + O$ first-order conditions plus O additional asset pricing conditions from the household problem. Meanwhile, we have $N + 2O$ unknowns, implying an exactly identified system of equations.

We can identify the markups $\{\boldsymbol{\mu}\}$ using the first-order condition of variable inputs, while $\{\boldsymbol{\tau}\}$ and $\{\boldsymbol{\zeta}\}$ can be jointly identified using the first-order condition of the capital inputs and the asset pricing equations. This forms the core of our identification and allows us to compute the capital wedges while controlling for heterogeneous markups and risk premia. This approach is therefore a generalization of the approaches focusing only on the firm side for the identification of wedges, which usually assume either no risk premia or homogeneous ones.

3.4 General Equilibrium

For every period $t \in [0, \infty)$, given exogenous discount factor shock Z_t, Hicks-neutral productivity z_{it}, markups $\{\boldsymbol{\mu}_{it}\}$, risk premia $\{\boldsymbol{\zeta}_{it}\}$, frictions $\{\boldsymbol{\tau}_{it}\}$, variable input prices $\{p_t^{\ell}\}$, and capital input prices $\{p_t^k\}$, a general equilibrium is set of output prices $\{p_{it}\}$ and quantities $\{q_{it}\}$, factor input choices $\{\ell_{it}\}$ and $\{k_{it}\}$, demands $\{c_{it}\}$, and final demand C_t such that: each producer minimizes its costs and charges the relevant markup on its marginal cost; household chooses consumption and savings to maximizes utility subject to the budget constraint; and the markets for all goods and factors clear.

Note that in the definition of equilibrium, the assumption that all factor prices are exogenous implicitly reflects a setting in which all variable inputs are the outcome of roundabout production. Alternatively, we could allow for an arbitrary number of factors to be directly supplied by the representative household—similar to labor—making their aggregate prices determined by market clearing. Although these represent distinct microfoundations, they would yield an isomorphic set of observables in our model. Thus, while adopting the alternative formulation would require a slight adjustment to the equilibrium definition for consistency, we assume roundabout production without loss of generality.

3.5 Micro-to-Macro Link

Here, we demonstrate how the divergence of the return on capital, R^k, from the risk-free rate, r, can be explained by (i) firm-level risk premia, (ii) firm-level mismeasurement of capital or frictions, (iii) firm-level markups, and (iv) firm-level fixed costs. We define the *measured* firm-level returns on capital as,

$$R^k_{it} \equiv \frac{p_{it} q_{it} - \boldsymbol{f}_{it} - \sum_{k^o \in \{k\}} \delta^{k^o} p^{k^o}_t k^o_{it} - \sum_{\ell^n \in \{\ell\}} p^{\ell^n}_t \ell^n_{it}}{\sum_{k^o \in \{k\}} p^{k^o}_t k^o_{it}}. \tag{12}$$

The above definition represents the measured firm-level return on capital, following the same approach used in the national accounts, as specified in equation (2). Specifically, the measured return on capital is defined as the ratio of output net of variable costs, fixed costs, and depreciation to the total capital stock.

Proposition 3 (Firm-Leve Return on Capital) *The measured firm-level return on capital can be decomposed into the sum of the true return on capital, $\mathcal{R}^k_{it}$, and the profit rate, π_{it}, as:*

$$R^k_{it} = \mathcal{R}^k_{it} + \pi_{it}, \tag{13}$$

where true return on capital is defined as follows:

$$\mathcal{R}^k_{it} \equiv r_t + \sum_{k^o \in \{k\}} \kappa^{k^o}_{it} \boldsymbol{\zeta}^{k^o}_{it} + \sum_{k^o \in \{k\}} \kappa^{k^o}_{it} \boldsymbol{\tau}^{k^o}_{it}, \tag{14}$$

which is simply the sum of the risk-free rate, the capital weighted risk premium and capital-specific wedges, and weights κ^{k^o} are the share of each capital type k^o within the firm. The profit rate is as:

$$\pi_{it} \equiv \frac{p_{it} q_{it}}{\sum_{k^o \in \{k\}} p^{k^o}_t k^o_{it}} \left(1 - \frac{\sum_{k^o \in \{k\}} \mathcal{E}_{k^o} + \sum_{\ell^n \in \{\ell\}} \mathcal{E}_{\ell^n}}{\boldsymbol{\mu}_{it}} \right) - \frac{\boldsymbol{f}_{it}}{\sum_{k^o \in \{k\}} p^{k^o}_t k^o_{it}}, \tag{15}$$

which represents the return from charging markups, net of the share attributed to fixed costs.

Proof. See Appendix B.4. ■

The fact that the measured return on capital equals the true one plus the profit rates stems from the fact that standard measurement assumes zero profits. Therefore, our methodology enables the distinction between movements in the true return on capital and movements in the return to firm ownership, as captured by profits.

To link these firm-level objects to the aggregate divergence of interest we start by the definition of measured aggregate capital return R^k presented in Section 2 that can be expressed as:

$$R_t^k = \frac{Q_t - \delta K_t - W_t L_t}{K_t}, \tag{16}$$

$$= \sum_{i\in\mathcal{I}} \omega_{it} \left(\frac{p_{it}q_{it} - \boldsymbol{f}_{it} - \sum_{k^o\in\{k\}} \delta^{k^o} p_t^{k^o} k_{it}^o - \sum_{\ell^n\in\{\ell^n\}} p_t^{\ell} \ell_{it}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}} \right) = \sum_{i\in\mathcal{I}} \omega_{it} R_{it}^k; \tag{17}$$

with $K_t \equiv \sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^o$ is aggregate capital, $Q_t - W_t L_t \equiv \sum_i (p_{it}q_{it} - \sum_{\ell^n\in\{\ell^n\}} p_t^{\ell^n} \ell_{it}^n - \boldsymbol{f}_{it})$ is total GDP net of the aggregate wage bill, $\delta K_t = \sum_{k^o\in\{k\}} \delta^{k^o} p_t^{k^o} k_{it}^o$ is total aggregate depreciation, and $\omega_{it} = \left(\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^o \right) / K_t$ is the share of firm-level caital share. Equations (16)-(17) demonstrate that the measured aggregate return on capital can be precisely represented as the capital-weighted average of the measured firm-level returns on capital.

Proposition 4 (Decomposition of the Return on Capital net of Risk-Free Rate) *The difference between the return on capital and the risk-free rate is given by:*

$$R_t^k - r_t = \underbrace{\sum_{i\in\mathcal{I}} \omega_{it} \left(\mathcal{R}_{it}^k - r_t \right)}_{\textit{True return on capital}} + \underbrace{\sum_{i\in\mathcal{I}} \omega_{it} \pi_{it}}_{\textit{Profits}}, \tag{18}$$

$$= \underbrace{\sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \omega_{it} \kappa_{it}^{k^o} \boldsymbol{\zeta}_{it}^{k^o}}_{\textit{Risk}} + \underbrace{\sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \omega_{it} \kappa_{it}^{k^o} \boldsymbol{\tau}_{it}^{k^o}}_{\textit{Capital-Wedges}} + \underbrace{\sum_{i\in\mathcal{I}} \omega_{it} \pi_{it}}_{\textit{Profits}}. \tag{19}$$

Proof. See Appendix B.5. ■

Equation (18) demonstrates that the divergence between the measured return on capital,

R^k, and the risk-free rate, r, can be decomposed into two components: the divergence between the capital-weighted average of firm-level true returns on capital, $\mathcal{R}^k_{it}$, and the risk-free rate, r, as well as the capital-weighted average profit rate.

Instead, equation (19) shows that the divergence between R^k and r can be decomposed into three aggregate components, each representing a capital-weighted average of the corresponding firm-level factors. Risk premia reflect a reduced willingness of investors to supply capital to firms, which forces firms to operate with less capital, resulting in higher returns on capital. Instead, capital-specific wedges appear in equation (19) as adjustments to the observable capital-specific revenue-based marginal product, allowing us to recover the true capital-specific revenue-based marginal product, as defined in equation (7).

Shifting to profit-related explanations, markups and fixed costs, as defined by the profit share in equation (15), influence the difference between the measured aggregate return on capital and the risk-free rate. This occurs because they represent deviations from the perfect competition assumption, which standard measurements in the literature rely on, as explained in Section 2. When this assumption does not hold, the rise in returns to firm ownership is incorrectly attributed to capital ownership. Therefore, while an increase in markups, by boosting profits, can contribute to the widening gap between R^k and r, a rise in fixed costs would have the opposite effect, as it implies a reduction in profits.

Over the past few decades, economies have undergone significant structural transformation, marked by the relative decline of sectors like manufacturing and agriculture and the growing dominance of services. This shift has important implications for the aggregate return on capital. For example, the rise of the data economy has given rise to superstar firms with high markups and potentially elevated profit margins. A reallocation of economic activity toward such firms may increase the aggregate return on capital due to rising profit rates. To understand this effect, we decompose changes in the return on capital into our three key components: risk, profits, and capital frictions.

Proposition 5 (Sector-Level Decomposition) *The effect of structural change–reallocation* *of economic activity across sectors–is,*

$$
\begin{aligned}
\Delta(R_t^k - r_t) = &\sum_s \sum_{k^o \in \{k\}} \omega_{s,t-1} \Delta \boldsymbol{\zeta}_{st}^{k^o} + \sum_s \sum_{k^o \in \{k\}} \Delta\omega_{s,t} \boldsymbol{\zeta}_{st-1}^{k^o} + \sum_s \sum_{k^o \in \{k\}} \Delta\omega_{s,t} \Delta \boldsymbol{\zeta}_{st}^{k^o} \\
&+ \sum_s \sum_{k^o \in \{k\}} \omega_{s,t-1} \Delta \boldsymbol{\tau}_{st}^{k^o} + \sum_s \sum_{k^o \in \{k\}} \Delta\omega_{s,t} \boldsymbol{\tau}_{st-1}^{k^o} + \sum_s \sum_{k^o \in \{k\}} \Delta\omega_{s,t} \Delta \boldsymbol{\tau}_{st}^{k^o} \\
&+ \sum_s \sum_{k^o \in \{k\}} \omega_{s,t-1} \Delta \pi_{st}^{k^o} + \sum_s \sum_{k^o \in \{k\}} \Delta\omega_{s,t} \pi_{st-1}^{k^o} + \sum_s \sum_{k^o \in \{k\}} \Delta\omega_{s,t} \Delta \pi_{st}^{k^o},
\end{aligned} \tag{20}
$$

where $\hat{\omega}_{it}$ is the firm capital share within a sector, $\boldsymbol{\zeta}_{st} \equiv \sum_{i\in s} \hat{\omega}_{it} \boldsymbol{\zeta}_{it}$, $\boldsymbol{\tau}_{st} \equiv \sum_{i\in s} \hat{\omega}_{it} \boldsymbol{\tau}_{it}^{adjusted}$, $\pi_{st} \equiv \sum_{i\in s} \hat{\omega}_{it} \pi_{it}^{adjusted}$, and $\omega_{st} \equiv \frac{\sum_{i\in s}\omega_{it}}{\sum_i \omega_{it}}$.

Proof. See Appendix B.5. ■

Equation (20) breaks down the growth in the $R_t^k - r_t$ into three components: Δwithin, Δbetween, and Δcross terms. The Δwithin component captures the portion of the change due to shifts in the sector-level wedges while holding sector-level weights constant. The Δbetween component captures the change due to shifts in sector-level weights, keeping the sector-level wedges constant. The Δcross term reflects the covariance between changes in weights and sector-level wedges. We combine the Δbetween and Δcross terms into a single Δreallocation component. In our final theoretical result, we characterize the response of aggregate return on capital to any firm-level shock.

Proposition 6 (Firm-Level Decomposition) *The response of the difference between return on capital and risk-free rate to a microeconomic shock can be written as:*

$$
\begin{aligned}
\Delta(R_t^k - r_t) = &\underbrace{\Delta\overline{\boldsymbol{\zeta}_t}}_{\text{Direct Effect}} + \underbrace{\Delta \sum_{i\in\mathcal{I}} \sum_{k^o \in \{k\}} \left(\boldsymbol{\zeta}_{it}^{k^o} - \overline{\boldsymbol{\zeta}}_t\right)\left(\omega_{it}\kappa_{it}^{k^o} - \overline{\omega_t \kappa_t}\right)}_{\text{Reallocation Effect}} \\
&+ \Delta\overline{\boldsymbol{\tau}}_t + \Delta \sum_{i\in\mathcal{I}} \sum_{k^o \in \{k\}} \left(\boldsymbol{\tau}_{it}^{k^o} - \overline{\boldsymbol{\tau}}_t\right)\left(\omega_{it}\kappa_{it}^{k^o} - \overline{\omega_t \kappa_t}\right) \\
&+ \Delta\overline{\pi_t} + \Delta \sum_{i\in\mathcal{I}} (\pi_{it} - \pi_t)(\omega_{it} - \overline{\omega_t}).
\end{aligned} \tag{21}
$$

where $\Delta\overline{\zeta_t}$, $\Delta\overline{\tau_t}$ *and* $\Delta\overline{\pi}_t$ *are changes in average risk permia, frictions and profits. While* $\overline{\omega_t \kappa_t} = \frac{1}{\mathcal{I}}\frac{1}{\mathcal{O}}\sum_{i\in\mathcal{I}}\sum_{k^o\in\{k\}}\omega_{it}\kappa_{it}^{k^o}$ *and* $\overline{\omega_t} = \frac{1}{\mathcal{I}}\sum\omega_{it}$ *are the average share of a specific capital type and the total capital across firms.*

Proof. See Appendix B.6. ■

Equation (21) highlights two effects of any microeconomic shock: a direct effect and a reallocation effect. For example, an increase in risk premia directly raises the return on capital. However, it may also induce a reallocation of capital across firms. For instance, higher risk premia may reduce the capital share of high risk premia firms, dampening the aggregate effect. As a result, the overall impact may be smaller than the direct effect alone. Overall, the firm-level equation (10), along with equations (5), (6), and (7), which are linked to the macro level through equation (19), provide a micro-to-macro methodology applicable to standard firm-level data sources for decomposing exactly the divergence between R^k and r into various potential competing explanations debated in the literature. The following sections detail how to implement this methodology and the results it yields.

4 Data, Variable Construction, and Measurement

4.1 Data Sources

Firm-level data. The primary data source is Compustat, a firm-level database that provides balance sheet information for all U.S. publicly traded firms covering the entire period of interest. The key advantage of using Compustat is its comprehensive coverage of the relevant time frame and a wide range of sectors. While publicly traded firms represent only a small fraction of the total number of firms, they are often among the largest in the economy, accounting for approximately 30% of U.S. employment Davis et al. (2006). More relevant to our focus on capital, the particularly right-skewed distribution of capital across firms makes these large firms highly informative for understanding aggregate capital, as noted by Crouzet and Mehrotra (2020). According to our calculations, they represent approximately 60 percent of total non-residential capital.

Aggregate risk factors data. We obtain data on aggregate risk factors from various sources. For the standard CAPM, the five-factors data from Hou et al. (2015) can be accessed at http://global-q.org/factors.html. Additionally, the five-factors, three-factors, and one-factor data from Fama and French (2023) are available on Kenneth French's website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/. Finally, when using the consumption CAPM, we employ a single-factor model that incorporates the consumption growth rate from the NIPA tables.

4.2 Definition and Construction of Variables

In this section, we provide a brief overview of how we define and compute the main variables of interest used in the empirical analysis. See Appendix C.1 for further details on the cleaning process and for summary statistics of our main variables.

4.2.1 Definition of Variables

While our theory is more general, we follow standard practice—due to limitations in the Compustat data and for comparability with the existing literature—by modeling a single capital input, a single variable input, and a single fixed cost, all defined in Section 4.2.2. We retain a general specification for firm-level demand but restrict the production technology to a Cobb-Douglas form with time-varying, sector-specific coefficients, as described in Section 4.3.1.

4.2.2 Construction of Variables

Capital Stock. We compute the total capital stock of a firm as a sum of tangible capital and intangible capital following Peters and Taylor (2017). The tangible capital is constructed using the perpetual inventory method:

$$k_{it}^{T} = (1 - 0.07)k_{it-1}^{T} + x_{it}^{T}, \tag{22}$$

where $x_{it}^{T} - 0.07k_{it-1}^{T} \equiv \texttt{ppent}_{it} - \texttt{ppent}_{it-1}$, and values are deflated. The initial capital stock is set as $k_{i0}^{T} = \texttt{ppegt}_{it}$. We measure intangible capital stock as the sum of three components, following best practices in corporate finance (Peters and Taylor, 2017; Ewens et al., 2024). The first component, knowledge capital, is capitalized R&D, defined as:

$$k_{it}^{KNWL} = (1 - \delta_{s(i)}^{KNWL})k_{it-1}^{KNWL} + \texttt{xrd}_{it}, \tag{23}$$

where sector-level depreciation rates, $\delta_{s(i)}^{KNWL}$, are from Ewens et al. (2024), and $k_{i0}^{KNWL} = 0$.[4] The second component, organizational capital, is constructed by capitalizing a portion of selling, general, and administrative expenses as follows:

$$k_{it}^{ORG} = (1 - 0.20)k_{it-1}^{ORG} + \gamma_{s(i)}\texttt{xsga}_{it}, \tag{24}$$

where $\gamma_{s(i)}^{ORG}$ is from Ewens et al. (2024), and $k_{i0}^{ORG} = 0$.The third component, balance sheet intangible capital net of goodwill, is defined as: $k_{it}^{BS} = \texttt{intano}_{it}$.[5] Total intangible capital is calculated as: $k_{it}^{I} = k_{it}^{KNWL} + k_{it}^{ORG} + k_{it}^{BS}$, and values are deflated. Total capital stock at the firm-level and total depreciation is defined as the is defined as

$$k_{it} = k_{it}^{I} + k_{it}^{T}, \quad \delta_{it} = \frac{k^{T}}{k_{it}}\delta^{T} + \frac{k^{I}}{k_{it}}\delta^{I}. \tag{25}$$

Output, and variable and fixed costs. The Compustat data include detailed firm-level financial statements, such as sales, input expenditures, capital stock, and industry classifications. We use sales, $\texttt{sale}_{it}$, to measure firm output, cost of goods sold, $\texttt{cogs}_{it}$, to capture variable input, and the non-capital fraction of selling, general, and administrative expenses, $(1 - \gamma_{s(i)}^{ORG})\texttt{xsga}_{it}$, to measure fixed costs. We deflate all variables to obtain their real values.

Relative prices. We measure the relative price of tangible capital, p_t^T, as the ratio of the tangible capital investment deflator to the GDP deflator, and the relative price of intangible capital, p_t^I, as the ratio of the intangible capital deflator to the GDP deflator.

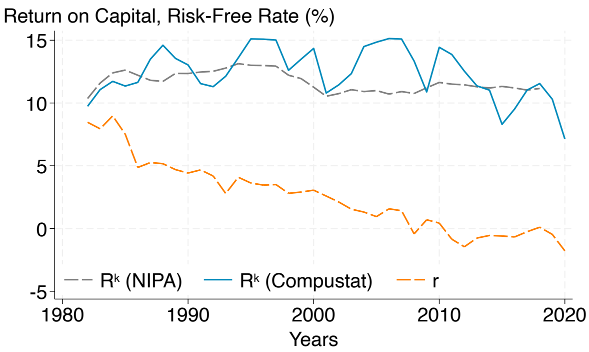
Firm-level and aggregate return on capital. We calculate the firm-level return on capital, R_{it}^{k}, using equation (12). To assess how closely the aggregate return on capital from Compustat aligns with national accounts, we aggregate the firm-level returns, weighting them by total capital: $R_t^k = \sum_i \omega_{it} R_{it}^k$, in accordance with equation (17).

Figure 2a displays the evolution of the return on capital from national accounts (NIPA) and firm-level data (Compustat), showing a strong correlation between the two. This is unsurprising, given the well-established fact that the distribution of capital is heavily skewed toward larger firms (Crouzet and Mehrotra, 2020), which are highly representative of aggre-

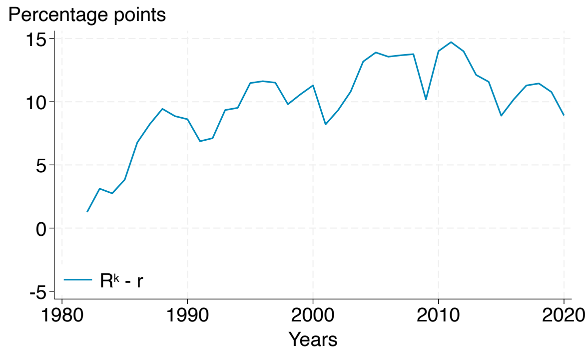
[4]We provide results with a different measure of initial capital stock as the ratio of the initial investment and the depreciation rate.

[5]This component is crucial to include, as it typically accounts for most software on the balance sheet, which is the fastest-growing segment of aggregate intangible capital according to BEA data. For a detailed discussion on software accounting standards, see Chiavari and Goraya (2024) and Aum and Shin (2024).

Figure 2: Comparison Between National Accounts and Compustat



(a) Return on Capital and Risk-Free Rate



(b) Return on Capital and Risk-Free Rate Gap

Note. Figure 2a illustrates the evolution of the aggregate return on capital from national accounts and the risk-free rate, as previously presented in Figure 1, alongside the aggregate return on capital from Compustat. Figure 2b highlights the aggregate divergence between the return on capital from Compustat and the risk-free rate.

gate movements; as mentioned before, based on our calculations, they account for approximately 60 percent of total non-residential capital. This confirms that Compustat is a reliable dataset for studying the divergence of the return on capital from the risk-free rate. Figure 2b quantifies this divergence, which grew from nearly zero in 1982 to almost 10 percentage points by 2019. Next, we examine the firm-level components from equation (19) that may have contributed to this divergence, assessing their quantitative explanatory power.

4.3 Firm-Level Measurements

4.3.1 Production Function Elasticities and Markups

This section outlines our baseline production function and markup measurement, describes the main findings, and presents a series of additional exercises we conduct to evaluate their robustness.

Production function and markup measurement. The two most prominent approaches in the literature for estimating firm-level production functions are the cost share approach and the control function approach. Although the former requires a measure of the user cost of capital—which depends on unobservable risk premia and capital frictions—the latter does not.[6] For this reason, we adopt the control function approach, specifically following the recent

[6]The literature often sidesteps this challenge by either assuming away risk premia and frictions or calibrating them. This is not feasible in our setting, as these are precisely the objects required to be estimated as inputs of

implementation by Ackerberg and De Loecker (2021).

A key challenge in implementing this approach with Compustat data is the absence of separate information on output prices and quantities. While this limitation is common across many datasets, it forces researchers to use revenue—rather than physical output quantities—on the right-hand side of production function estimations, as recently emphasized by Bond et al. (2021). Although our model is sufficiently general to accommodate flexible demand and production function assumptions, this empirical constraint necessitates the imposition of additional identifying restrictions.

To address this challenge, we follow Ackerberg and De Loecker (2021), who provide a recent framework for estimating production functions under imperfect competition (i.e., variable markups) and in settings where only revenue data are available. Consistent with their approach, we assume a sector-specific, time-varying Cobb-Douglas production function at the 2-digit NAICS level. On the demand side, we consider three market structures: the Homogeneous Goods Quantity-Setting Model, the Logit Nash-Bertrand Model, and the Nested Logit Nash-Bertrand Model. These assumptions allow us to use the revenues of competing firms within an industry as a sufficient statistic to control for demand variation—and thus for variable markups—effectively restoring the scalar unobservability condition required for first-stage estimation. In addition, this approach introduces demand-side variation similar to that of an oligopoly instrument, helping to overcome the non-identification issues associated with gross-output production functions, as highlighted by Gandhi et al. (2020).

We embed this first-stage approach in a system GMM estimation framework, using moment conditions from Blundell and Bond (1998), which accommodate firm-level fixed effects in productivity. Although our empirical strategy is exact under the assumed demand structures, there is no guarantee that these assumptions fully reflect the true nature of competition in the data. However, because our methodology requires as input changes over time as highlighted in equation (21), we emphasize that accurately capturing trends in output elasticities and markups is more important than identifying levels. In richer settings where price data are observed, De Ridder et al. (2024) shows that using revenue instead of output quantity data affects the levels of estimated elasticities but still captures trends well–which is our primary concern.

Finally, with time-varying sector-level estimates of production function elasticities in

our theory.

hand, we construct markups following De Loecker and Warzynski (2012), which coincides with equation (5). This approach recovers firm-level markups as the product of the elasticity with respect to the variable input and the ratio of revenues to variable input expenditures.

Baseline estimates. Appendix C.2 presents the evolution of the estimated elasticities of the production function and markups. In line with De Loecker et al. (2020), we observe a steady increase in the elasticity of capital over time, while the elasticities of variable costs have declined modestly. Additionally, we find a steady increase in average markups, which aligns with the extensive literature documenting this development, as reviewed recently by Syverson (2024).

Robustness exercises. We demonstrate in Appendix C.2 that our estimates of the elasticities of the production function and markups are robust across a variety of different methodologies.

First, we sidestep the first stage of the production function estimation by collapsing it to a standard system GMM estimator in the spirit of Blundell and Bond (1998). Alternatively, we retain the original first-stage procedure and instead modify the second stage by applying a set of moment conditions, following the approach of Ackerberg et al. (2015). As a further robustness check, we employ the method developed by Collard-Wexler and De Loecker (2021), which accounts for classical measurement error in capital stock. Across these approaches, we find that the estimated output elasticities and markups exhibit trends consistent with those obtained from our baseline method.

Additionally, we consider an alternative markup measure to that proposed by De Loecker and Warzynski (2012). Specifically, we adopt the accounting profit approach from Baqaee and Farhi (2020), calculating markups as the ratio of sales to total costs, including both variable and fixed overhead costs. This alternative yields markup estimates broadly in line with those derived from our baseline method.

4.3.2 Risk Premium

Equation (11) represents a generalization of the standard CAPM and naturally leads to a two-stage procedure for estimating risk premia, consistent with common practice in the empirical asset pricing literature.

First stage. The first step involves estimating $\boldsymbol{\beta}_{it}$, which captures a firm's exposure to movements in the stochastic discount factor. Since this exposure is defined as the covariance between $\widetilde{MRPK}$ and M divided by the variance of M, it can be estimated using a simple firm-specific OLS regression, given by:

$$\widetilde{MRPK}_{it} = \alpha_{i\tau} + \boldsymbol{\beta}_{i\tau} M_t + \varepsilon_{it}. \tag{26}$$

We estimate these regressions using backward-looking rolling windows of N_τ years. Specifically, for each year τ, we use the data from the period $t \in [\tau - N_\tau, \tau]$.

Second stage. Given a time-varying firm-level estimate $\boldsymbol{\beta}^j_{it}$, we can estimate the aggregate price of risk $\boldsymbol{\lambda}_t$ using equation (11). In practice, this implies to estimate the following cross-sectional regression in each year:

$$\widetilde{MRPK}_{it} - r_{t-1} - \delta_{it} = \alpha_{s(i)t} + p(age_{it}; \phi_t) + \boldsymbol{\beta}_{it}\boldsymbol{\lambda}_t + \varepsilon_{it}. \tag{27}$$

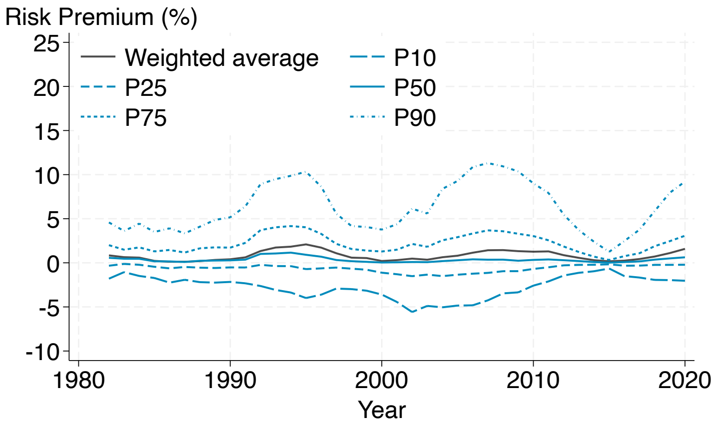
We use a combination of sector-time fixed effects and a third-degree polynomial in age to control for firm-level capital wedges, as well as time-varying capital gains present in equation (11). Although this step is not necessary for identification—since by construction, capital wedges and capital gains are uncorrelated with the risk premium—it helps to improve precision in small samples. Finally, the time-varying firm-level risk premium is measured as a 3-year moving average of $\boldsymbol{\zeta}_{it} = \boldsymbol{\beta}_{it}\boldsymbol{\lambda}_t$.

Empirical implementation and results. We implement this two-stage procedure by approximating the stochastic discount factor, M_t, using the five-factor model from Hou et al. (2015), which has been shown to track cross-sectional firm-level variation well and was recently applied in a similar context by David et al. (2022). These factors include (i) the market return, (ii) the return on a portfolio that is long in small firms and short in large firms, (iii) the return on a portfolio that is long in low-investment firms and short in high-investment firms, (iv) the return on a portfolio that is long in high-profitability (return on equity) firms and short in low-profitability firms, and (v) the return on a portfolio that is long in firms with high expected 1-year-ahead investment-to-assets changes and short in firms with low ones.

Figure 3 illustrates the evolution of estimated firm-level risk premia.[7] It highlights the 10th, 25th, 50th, 75th, and 90th percentiles, along with the capital-weighted average, which

[7] Given our longer-term focus, we apply a 5-year moving average to firm-level risk premia to smooth out excess volatility.

Figure 3: Evolution of the Risk Premium Distribution



Note. Figure 3 shows the evolution of various moments in the distribution of firm-level risk premia from 1982 to 2019, specifically reporting the 10th, 25th, 50th, 75th, and 90th percentiles, as well as the capital-weighted average.

serves as the input for equation (19). We find that most firms exhibit modest risk premia of approximately 2% on average, while the overall distribution is right-skewed, with a long right tail of firms facing substantially higher risk premia. Moreover, while the distribution of risk premia fluctuates significantly over time—widening notably during recessions—its average has remained remarkably stable over the past four decades.

Appendix C.3 validates our firm-level estimates of risk premia. Specifically, we show that firms with higher risk premia tend to exhibit higher returns on equity, lower levels of both tangible and intangible capital, and a lower capital-to-variable-costs ratio for both types of capital, as expected. Moreover, consistent with the findings of David et al. (2022), we observe that sectors with more dispersed risk premia also display greater dispersion in revenue-based marginal products of tangible capital, intangible capital, and revenue productivity.

Moreover, in Appendix C.3, we demonstrate that alternative factor models commonly used in the literature, such as the Fama and French (2023) 5-factor, 3-factor, and 1-factor models, as well as the consumption CAPM, produce a quantitatively similar evolution of the average capital-weighted risk premium over time when compared to the Hou et al. (2015) model. With these validations in place, we proceed to discuss how these estimates align with the existing evidence on risk premia in the literature.

Relation with existing evidence. Our estimated risk premium is consistent with a broad body of evidence and the asset pricing literature. Common risk indicators, such as the

VIX Index, the SKEW Index, the spread between the Fed Funds Rate and the Three-Month Treasury Bill Rate used in Drechsler et al. (2018), the spread between risky and safe assets in Jordà et al. (2019), the Excess Bond Premium from Gilchrist and Zakrajšek (2012), the Economic Policy Uncertainty Index by Baker et al. (2016), Robert Shiller's CAPE Ratio, the Chicago Fed's NFCI risk subindex, the financial uncertainty index of Jurado et al. (2015), the risk appetite index of Bauer et al. (2023), the risk aversion index of Bekaert et al. (2022), and the variance risk premium from Bekaert and Hoerova (2014), show no upward trend over time.

Moreover, several papers in asset pricing have estimated the risk premium over time.[8] Campbell and Thompson (2008), Lettau et al. (2008), Avdis and Wachter (2017) find no rise in the risk premium since the early 1980s, being either constant or declining. Jagannathan et al. (2001) using a Gordon-like stock valuation model finds similar results. Martin (2017) reports a risk premium that has been constant since the mid-1990s. Similarly, Gagliardini et al. (2016), using a time-varying cross-sectional risk premium estimator, finds a stable risk premium over time. Gormsen and Huber (2023) estimate a constant perceived risk premium since the early 2000s, and Duarte and Rosa (2015), in their review of the asset pricing literature, show that risk premium estimates are generally either constant or at times declining.

An exception in the literature is Farhi and Gourio (2018), who argue that while the risk premium remained stable during the 1980s and 1990s, it began rising in the 2000s, particularly around the Great Recession. Although this is broadly consistent with the sharp increase we capture during the Great Recession, the fact that our estimates show a subsequent reversal and no sustained long-run trend underscores the importance of controlling for factors beyond risk premia—such as capital wedges and markups—that may drive the divergence between the return on capital and the risk-free rate and act as confounding forces in this relationship.

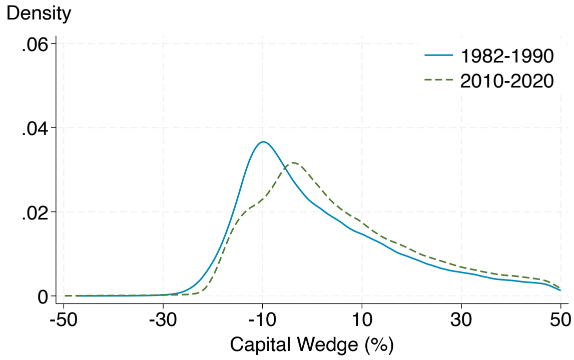
4.3.3 Capital Wedges

With firm-level measures of markups and risk premia in hand, we calculate capital wedges τ using equation (6). Figure 4a summarizes the distribution of capital wedges across different periods. Two key observations emerge: first, there is substantial dispersion in the capital wedge, ranging from -30 percent to +50 percent; second, this dispersion has modestly in-

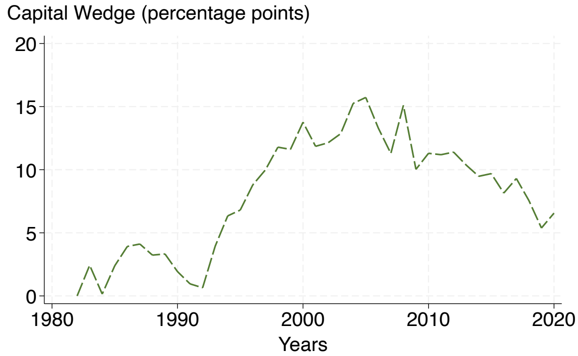
[8] Most of the existing asset pricing literature has concentrated on the equity risk premium. However, an important contribution by David et al. (2022) demonstrates that equity risk premia and capital risk premia are closely related and proportional, meaning that movements in one directly inform the other.

creased over time. Figure 4b shows the evolution of the aggregate capital wedge over time, measured by $\sum_i \omega_i \boldsymbol{\tau}_i$ in equation (19). The trend is clear: the aggregate capital wedge has been rising, reaching a level approximately 5 percentage points higher in 2020 compared to the 1980s.

Figure 4: Capital Wedge Distribution and Evolution



(a) Tangible Capital Wedge Distribution



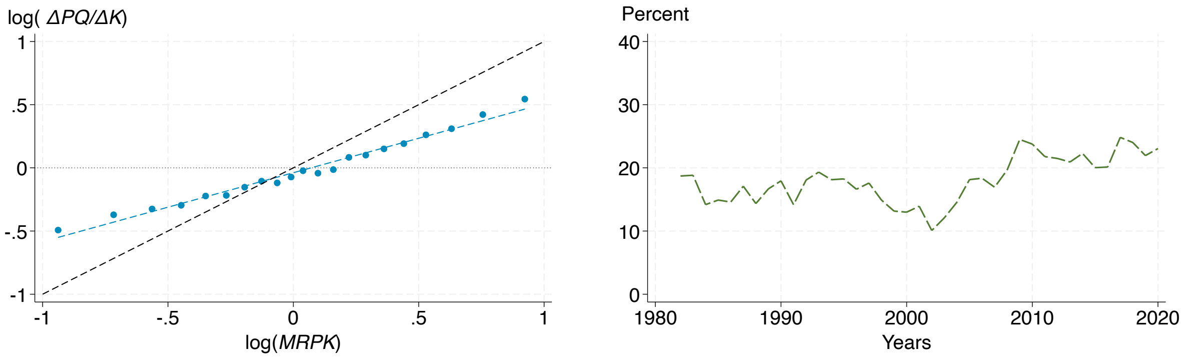
(b) Capital Wedge Evolution

Note. Figures 4a display the distribution of firm-level $\boldsymbol{\tau}_i$ for the periods 1982-1990 and 2010-2019. Figure 4b illustrates the evolution of the cumulative change in the weighted average of the capita-specific wedge.

Next, we investigate the nature of the estimated wedges. Specifically, we assess the roles of mismeasurement and frictions—such as financial constraints and adjustment costs—in explaining the rise of the capital wedge over time. Since the wedge may reflect either frictions or mismeasurement, we begin by quantifying the extent of mismeasurement in the observed wedges. We then net out the estimated mismeasurement and interpret the residual component as reflecting frictions, which we validate using standard proxies commonly employed in the empirical literature.

Capital mismeasurement. We begin by assessing the potential extent of measurement error in our estimation of the capital wedge using a simple approach proposed by Bai et al. (2024). This method leverages the definition of the revenue-based marginal product of capital as the change in output relative to the change in capital input, i.e., $\widetilde{MRPK} \approx \Delta pq/\Delta k$. This differencing not only provides a complementary measure of the revenue-based marginal product but also helps eliminate persistent measurement error, as suggested by Bai et al. (2024). Comparing our baseline measure with this alternative offers a straightforward diagnostic of potential measurement error in each component. Specifically, in the absence of measurement error, the two measures should be perfectly correlated. In contrast, if the original variation arises entirely from measurement error, the two measures should be uncorrelated.

Figure 5: Measurement Error in Revenue-Based Marginal Product of Both Capital



(a) Alternative Measures of MRPK **(b) Evolution of Mismeasurement**

Note. Figure 5a illustrates the relationship between our baseline measure of observed revenue-based marginal product and the measure constructed using the first differences of sales over capital. Both variables are deviations from sector time averages. The blue dots represent the relationship between $\log(MRPK)$ and $\log(\Delta PQ/\Delta K)$, with each point corresponding to one of the 25 percentiles. The dotted line indicates the best-fit line. Figure 5b presents the evolution of measurement error in the capital wedge, measured as the variance of $\log \beta_x^j$ to the variance of $\log \widetilde{MRPK}$ as in Bils et al. (2021).

Figure 5a displays the correlation between our baseline measure and the alternative measure constructed using first differences. Overall, we find that the two measures are highly correlated, indicating that measurement error accounts for only a portion of the observed variation in $\boldsymbol{\tau}$. Specifically, the alternative measure explains approximately 60% of the variation in $\log \widetilde{MRPK}$, suggesting that our baseline measure largely reflects underlying economic factors, though it still contains a nontrivial degree of measurement error.

To quantify and isolate the measurement error, we adopt the approach from the seminal work of Bils et al. (2021), as implemented in David and Venkateswaran (2019) and Bai et al. (2024). This method allows us to estimate the extent of additive measurement error by estimating the following regression:

$$\Delta \log p_{it} q_{it} = \alpha_\kappa + \beta_\kappa \log \Delta k_{it} + \varepsilon_{it}, \tag{28}$$

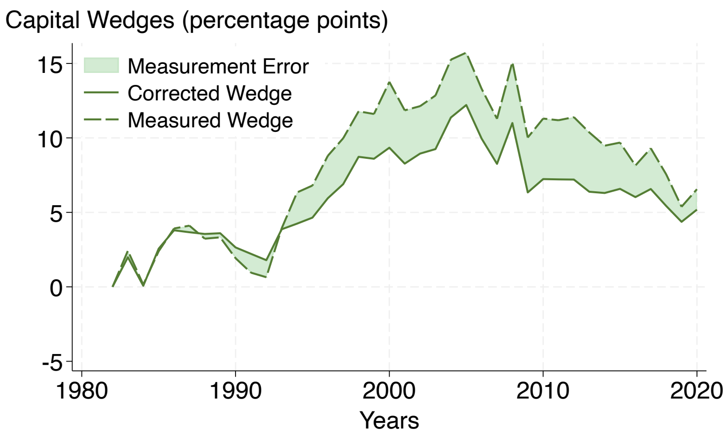
Here, $\Delta \log pq$ and $\Delta \log k$ represent the log changes in sales and capital, respectively, and κ denotes the decile of $\widetilde{MRPK}$. The key parameter of interest is the coefficient β_k. Intuitively, if the observed deviations in $\widetilde{MRPK}$ are primarily driven by additive measurement error, then firms with high observed $\widetilde{MRPK}$ should exhibit a lower elasticity of sales with respect to capital. Bils et al. (2021) show that, under certain assumptions, this coefficient identifies the exact extent of additive measurement error, allowing for the construction of a corrected measure:

$$\log \widehat{MRPK} = \log \widetilde{MRPK} + \log \beta_{\kappa}. \tag{29}$$

Hence, the ratio of the variance of $\log \beta_x$ to the variance of $\log \widetilde{MRPK}$ provides an estimate of the extent of measurement error in capital-specific wedges. Figure 5b presents our results. Several key observations emerge. First, capital wedges exhibit substantial measurement error—up to 20%. Second, measurement error has slight increase, particularly toward the end of the sample period. This finding is consistent with the results of Bils et al. (2021), who show—using data from the U.S. Annual Survey of Manufacturers in the Longitudinal Research Database—that measurement error in revenue-based total factor productivity has increased over time.

Finally, using these estimates of measurement error, we construct an adjusted wedge, $\boldsymbol{\tau}^{\text{adjusted}}$, purged of measurement error. Figure 6 shows its evolution alongside that of the original wedge, $\boldsymbol{\tau}$. We find that measurement error contributed to the rise in capital wedges, but not enough to account for the overall increase. The adjusted wedge, $\boldsymbol{\tau}^{\text{adjusted}}$, still rose by approximately 5 percent over the past four decades, indicating that the underlying trend remains substantial even after accounting for mismeasurement.

Figure 6: Role of Frictions and Measurement Error



Note. Figure 5a presents the evolution of the capital wedge over time. The shaded area is the proportion of the measurement error. The $\boldsymbol{\tau}$ as the dotted line and $\boldsymbol{\tau}^{\text{adjusted}}$ as the solid line.

Relationship between capital wedge and frictions. Here, we present suggestive evidence that the adjusted capital wedge, $\boldsymbol{\tau}_{it}^{\text{adjusted}}$, captures frictions such as financial frictions and adjustment costs. To do so, we collect several proxies for firm-level frictions commonly used in the literature.

These include liquidity, measured as the ratio of cash and short-term investments to total assets; leverage, defined as total debt over total assets; and the financial constraint indices from Hoberg and Maksimovic (2015) that are expanded to recent year by Linn and Weagley (2024), which capture debt- and equity-based financial constraints. The debt-based index identifies firms likely to delay investment due to liquidity issues, while the equity-based index captures both (a) firms at risk of delaying investment for liquidity reasons and (b) firms planning to issue equity. We also include average Tobin's Q, defined as the ratio of market value to total capital, as a proxy for investment frictions. Finally, we consider the investment rate in intangible capital, motivated by recent literature showing that intangible investment is highly frictional due to both higher adjustment costs and the lower collateralizability of intangible assets.

We estimate the following regression model:

$$\boldsymbol{\tau}_{it}^{\text{adjusted}} = \underbrace{g(\Upsilon_{it})}_{\text{Proxies for frictions}} + \varepsilon_{it}, \tag{30}$$

where $\boldsymbol{\tau}_{it}^{\text{adjusted}}$ denotes the measurement-error-adjusted capital wedge, and Υ_{it} represents a set of observable firm-level proxies for financial and investment frictions.

The results, reported in Table 1, indicate that the adjusted capital wedge is positively associated with debt-based measures of financial constraints, average Tobin's Q, and the rate of investment in intangible capital. In contrast, it is negatively associated with equity-based constraint indices. One interpretation of these findings is that firms with higher capital wedges are more likely to face barriers to debt financing, while their access to equity markets appears relatively less constrained.

5 The Micro-Anatomy of the Divergence

This section presents the main empirical results of the paper. First, it reports the decomposition of the divergence between the return on capital and the risk-free rate, as outlined in Section 3. Second, it examines the contribution of changes in sectoral composition, along with the role of firm-level dynamics and within-sector reallocation, in driving this divergence.

Table 1: Relationship between Capital Wedge and Frictions

Financial and investment constraints proxies	$\tau_{it}^{\text{adjusted}}$ (1)	$\tau_{it}^{\text{adjusted}}$ (2)
Liquidity	0.016	-0.055***
	(0.023)	(0.012)
Average Tobin's Q	0.001**	0.006***
	(0.001)	(0.002)
Leverage	0.000	0.000
	(0.000)	(0.000)
Text-based proxies by Hoberg and Maksimovic (2015)		
Debt-based Constraints	0.017***	0.017***
	(0.004)	(0.003)
Equity-based Constraints	-0.023***	-0.018***
	(0.005)	(0.003)
Intangibles		
Intangible Investment Rate	0.131***	0.041***
	(0.024)	(0.015)

Note. This table provides regression coefficients with $\tau_{it}^{\text{adjusted}}$ as dependent variable and proxies of frictions as independent variables. It controls for firm and year-fixed effects. In Column 1, we have all observations, while in Column 2, we keep firms that are older than 3 years. Standard errors are clustered at the firm level and reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively

5.1 The Macroeconomic Drivers

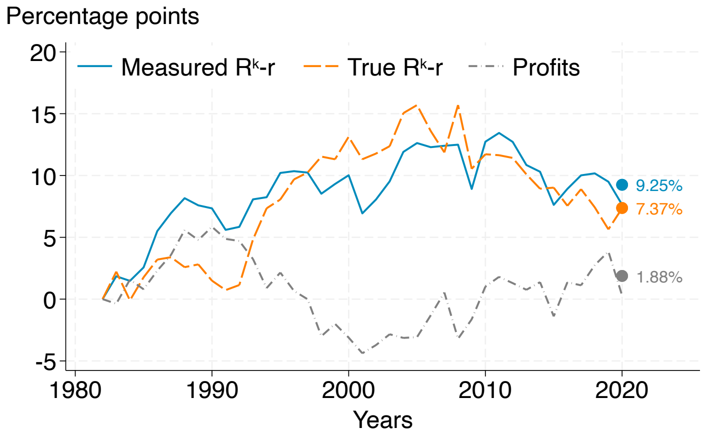
Here, after measuring all the necessary firm-level variables required by our theory, we analyze the joint contribution of each narrative associated with the divergence between the measured aggregate return on capital and the risk-free rate. To achieve this, we first isolate the contribution of profits from the underlying dynamics of the true return on capital, using equation (18).

Figure 7 illustrates the evolution of the divergence between the measured return on capital and the risk-free rate, along with the evolution of each component as calculated in equation (18), and summarizes our first finding:

Fact 1: *Since 1982, 20% of the divergence between the return on capital and the risk-free rate is due to profits, while the rest is due to the missing decline in the true return on capital.*

Figure 7 shows that over time, the gap between the return on capital and the risk-free rate has widened, averaging 9.25 percentage points between 2015 and 2020. It also presents the counterfactual evolution of this gap, illustrating scenarios in which only profits or only

Figure 7: The Role of Profits and True Return on Capital



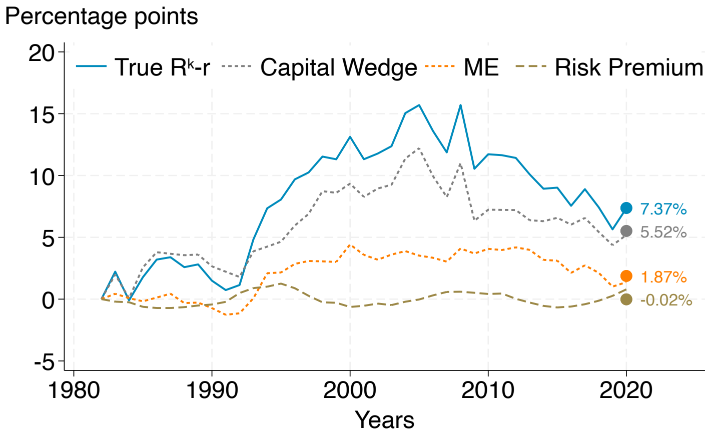
Note. Figure 7 presents the decomposition based on equation (18) for the period 1982 to 2019. It illustrates the evolution of the gap between the return on capital and the risk-free rate (solid blue line), alongside the contributions of profits (dash-dotted green line) and the true return on capital (long-dashed orange line). The dots highlight the average between 2015-2020 for every variable.

the true return on capital have changed over time. We find that if profits were the sole driver of the divergence, the gap would have increased by approximately 1.88 percentage points, accounting for 20 percent of the divergence. In contrast, if only the true return on capital were responsible, the divergence would have reached just 7.37 percentage points, or 80 percent. Therefore, if profits were correctly accounted for, the measured gap between the return on capital and the risk-free rate would have risen less over time.

We emphasize that the finding that the rise in profits explains a substantial portion of the divergence between the aggregate measured return on capital and the risk-free rate aligns with the seminal work of De Loecker et al. (2020), which documents a notable increase in market power. The extensive literature that followed has confirmed the broad notion of rising markups, though the magnitude of this increase varies depending on how markups are measured and aggregated. This body of work has been recently reviewed and summarized by Syverson (2024). Appendix D.1 demonstrates that decomposing the role of profits into markups and fixed costs reveals that both factors contributed roughly equally to the widening gap between the return on capital and the risk-free rate.

Next, we examine why the true return on capital has diverged from the risk-free rate. We leverage the decomposition in equation (19) to shed light on which of the possible explanations—risk premium or frictions related to capital—best explains this divergence. Figure 8 presents the results from this decomposition. Particularly, it plots the divergence between the

Figure 8: The Drivers of The True Return on Capital and Risk-Free Rate Divergence



Note. Figure 8 presents the decomposition based on equation (19) for the period 1982 to 2019. It illustrates the evolution of the gap between the true return on capital and the risk-free rate (solid blue line), alongside the contributions of frictions associated to intangible capital (dash-dotted green line), frictions associated to tangible capital (long-dashed orange line), and the risk premium (dashed brown line). The dots highlight the average between 2015-2020 for every variable.

true return on capital and the risk-free rate, along with the evolution of each component as calculated in equation (19), and summarizes our second finding:

Fact 2: *Contrary to previous findings, the rising frictions associated with capital–rather than the risk premium, which has remained relatively stable over time–have been the main force behind the divergence between the true return on capital and the risk-free rate.*

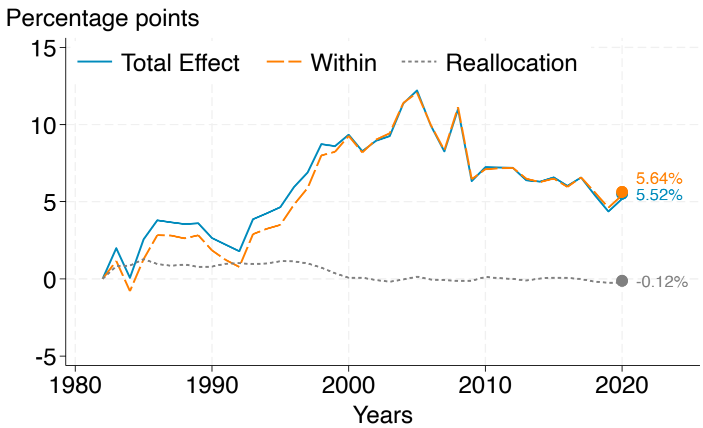
Figure 8 illustrates that frictions associated with capital have been rising, contributing to the divergence between the true return on capital and the risk-free rate. However, the rise in frictions is quantitatively much larger, predicting a divergence of 5.52 percentage points. In contrast to the previous consensus (e.g., Farhi and Gourio, 2018), we find that risk premia's contribution is only -0.02 percentage points to the overall divergence of 9.25 percentage points, consistent with extensive asset pricing literature and evident in many risk proxies reviewed in Section 4.3.2. Therefore, to understand why the gap between the true return on capital and the risk-free rate has persisted over time, it is essential to focus on the significant rise in frictions related to capital, which we will explore in greater detail in the following sections. Appendix D.2 shows that the results presented in Figure 7 are robust to the use of different production functions, measures of markups, and risk premiums, as discussed in Sections 4.3.1 and 4.3.2.

One advantage of our methodology is that it ties the various forces driving the divergence between the return on capital and the risk-free rate to their micro-level sources. This allows us to investigate the roles that sectors, firms, reallocation, and new cohorts have played in shaping this divergence. We explore these factors in the following sections.

5.2 The Role of Sectors

This section explores the role of sectors and their changing importance in the aggregate economy over time in driving the gap between the true return on capital and the risk-free rate. We focus specifically on the frictions associated with capital, as they are the most important driver of the divergence. Appendix D.3 provides a similar analysis for the other components outlined in equation (20). Figure 9 presents the results of the sectoral decomposition described in equation (20) for the period from 1982 to 2019 and summarizes our third finding:

Figure 9: Sectoral Decomposition of the Return on Capital and Risk-Free Rate Gap



Note. Figure 9 illustrates the results of the sectoral decomposition in equation (20) for the period from 1982 to 2019. The solid blue line represents the evolution of the capital wedge, while the long dashed orange line depicts the evolution of the Δwithin component. The short dashed grey line shows the evolution of the Δreallocation component. The dots highlight the average between 2015-2020 for every variable.

Fact 3: *Changes in the sectoral composition of the U.S. economy played no role in the rise of frictions associated to capital.*

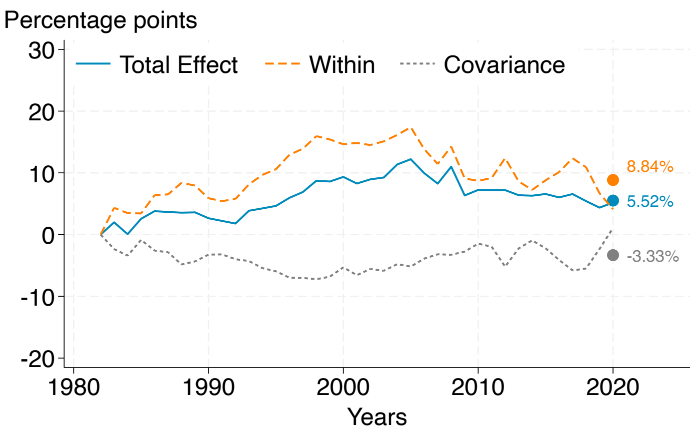
We find that the Δwithin component accounts for the entire divergence between the return on capital and the risk-free rate, while the Δreallocation component contributes nothing to this divergence. This holds true even when examining each part of the Δreallocation component separately, namely the Δbetween and Δcross term components, both of which remain very small throughout the analysis period. Appendix D.3 confirms that this is also the case when considering the other individual component contributing to the divergence in the true return on capital, specifically the risk premium.

Overall, our findings suggest that while the U.S. economy has experienced substantial deindustrialization over the analysis period, characterized by a continuous shrinkage of its manufacturing sector and a rise in services, the sources of the widening gap between the true return on capital and the risk-free rate, and in particular of the rise in frictions associated to capital must be sought elsewhere. In fact, what Figure 9 indicates is that, to uncover the root causes of this divergence, we must focus on within-sector forces. This sets the stage for our next section, which investigates the roles of firms, reallocation among them, and the changing nature of newer cohorts of firms.

5.3 The Role of Firms and the Changing Nature of Cohorts

This section delves into the role of firm dynamics and reallocation among firms, on the rise in frictions related to intangible capital. Similar to the previous section, Appendix D.4 offers a comparable analysis for the other components outlined in equation (21). In Figure 10, we find that the majority of the rise in the capital wedge is driven by the within effect–the average wedge has increased over time. This suggests that firms are increasingly facing higher frictions in accessing capital.

Figure 10: Firm-Level Decomposition of the Adjusted Capital Wedge



Note. Figure 10 illustrates the results of the firm-level decomposition in equation (21) for the period from 1982 to 2020. The solid blue line represents the evolution of the adjusted capital wedge, while the long dashed orange line depicts the evolution of the within component. The short dashed grey line shows the evolution of the covariance component. The dots highlight the average between 2015-2020 for every variable.

Next, we decompose the "within" effect into a cohort effect and a decade effect using the regression

$$\tau_{it}^{\text{adjusted}} = \underbrace{\mathbb{Y}_{d(i,t)}}_{\text{Decade effects}} + \underbrace{\mathbb{C}_{c(i,t)}}_{\text{Cohort effects}} + \varepsilon_{it}. \tag{31}$$

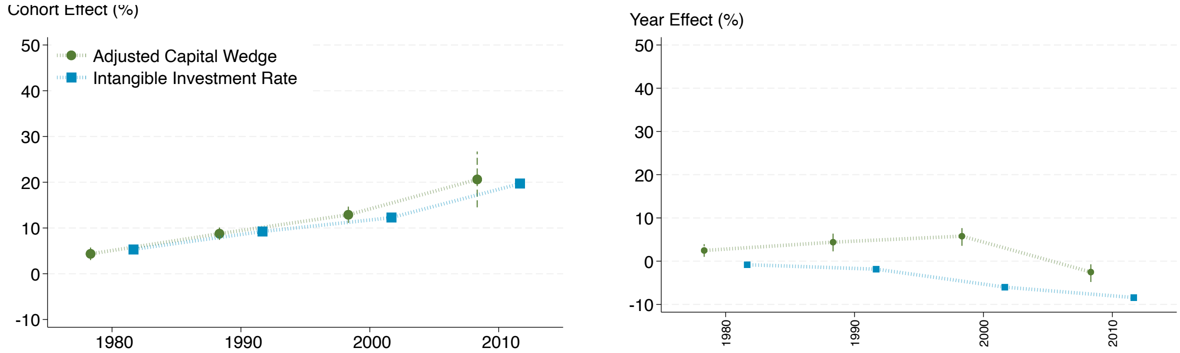
Figures 11a and 11b plot the estimated cohort and year dummies separately. We find that the recent rise in the capital wedge is driven almost entirely by newer cohorts of firms, whereas the year effect actually trends downward after its early-2000s peak. These results are robust to alternative cohort definitions (see Appendix D.4).

What makes these newer cohorts so different? A growing literature argues that modern firms expand by investing heavily in intangible assets—what Brynjolfsson et al. (2008) dub "scale without mass." However, the intangible capital investment process is more frictional. The intangible capital cannot be fully pledged as collateral, so firms that depend on it face tighter financing constraints (Caggese and Pérez-Orive, 2022; Falato et al., 2022; Bøler et al., 2023; Zhang, 2024) and incur substantially higher adjustment costs (Peters and Taylor, 2017; Belo et al., 2022; Cloyne et al., 2022; Crouzet and Eberly, 2023; Chiavari and Goraya, 2024). Consistent with these frictions, our data show a strong positive correlation between the capital wedge and intangible-investment intensity. Moreover, as Figures 11a–11b demonstrate, newer cohorts are markedly more intangible-intensive than older ones. We therefore interpret our cohort-effect findings as evidence that the rise in the capital wedge reflects the increasing prevalence of intangible-intensive firms that are subject to greater financing and adjustment frictions.

Fact 4: *The increase in the capital frictions is a within-firm phenomenon associated with newer intangible-intensive cohorts of firms.*

We conclude by emphasizing, as mentioned earlier, that since age in Compustat represents years since incorporation in the dataset, young firms should not be mistaken for new firms. Instead, they should more accurately be understood as newer large firms. As discussed, due to the highly skewed distribution of capital toward these firms, they are highly informative of aggregate capital movements. Our findings highlight that these markedly different new cohorts of large firms, with their initially higher intangible capital intensity, are central to the aggregate rise in frictions associated with intangible capital, and consequently, to the

Figure 11: Unpacking the Rise of Capital Frictions



(a) Cohort effects **(b) Decade Effects**

Note. Figures 11a, and 11b illustrate the evolution of the year effect and cohort effect as estimated using equation 31. The cohorts are grouped into firms born within 5 years. The intangible investment share is computed as a ratio of intangible investment and capital stock.

divergence between the true return on capital and the risk-free rate. In what follows, we study the aggregate implications of these frictions by imposing some structure on the generalized framework presented above.

6 Aggregate Implications

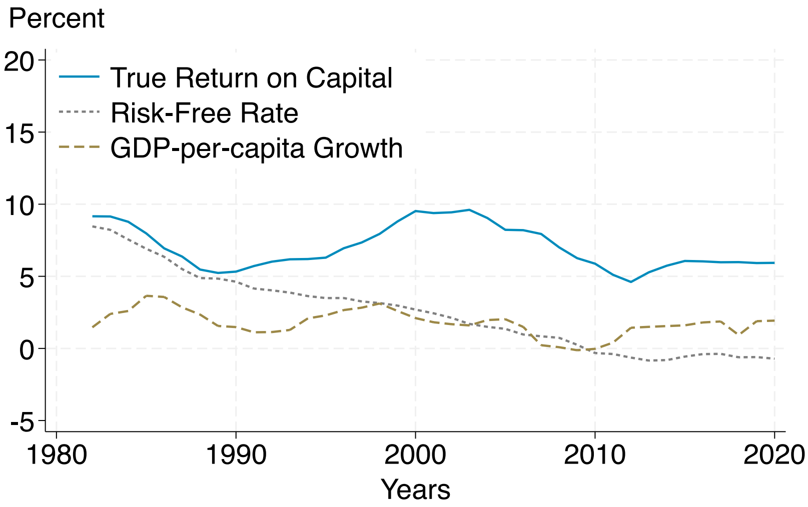
This section presents two aggregate implications. Section 6.1 provides the evolution of the true aggregate return on capital. Section 6.2 analyzes the effects of excess dispersion in return on capital on allocative efficiency.

6.1 True Return on Capital

Using the profit and measurement-error estimates from the previous section, Figure 12 shows the evolution of the true return on capital—defined as the measured return net of profits and measurement error. In contrast to the measured return, the true return has clearly trended downward, falling from approximately 9 percent at the beginning of the period to about 6 percent by 2020.

Despite this decline, the estimated return on capital remains above key benchmarks: it exceeds the average U.S. GDP per capita growth rate, which has remained below 2 percent over the past decade, and it is well above the risk-free rate, which has hovered near zero since the Great Recession. These findings confirm the observation by Piketty (2014) that capital

Figure 12: True Return on Capital: 1982-2020



Note. Figures 12 illustrate the evolution of the true return on capital–that is, the measured return net of profits and measurement error–, the risk-free rate, and the growth rate of GDP per-capita. These series are smoothed using a 5-year rolling window.

returns have exceeded economic growth—and, at least in theory, wage growth—and reinforce the concern raised by Reis (2022) regarding the dilemma faced by central banks in choosing whether to anchor policy rates to the risk-free rate or to the return on capital.

6.2 Excess Dispersion, Allocative Efficiency, and Aggregate Productivity

A nonzero value of $\boldsymbol{\tau}_{it}^{\text{adjusted}}$ suggests that there is excess dispersion in the return to capital even after accounting for measurement error. indicates excess dispersion in the return to capital, even after correcting for measurement error. This lack of return equalization across firms implies allocative inefficiency. Since the cost of capital is defined as the sum of the true return and the depreciation rate, $\mathcal{C}_{it}^{k} \equiv \mathcal{R}_{it}^{k} + \delta_{it}$, dispersion in $\mathcal{R}_{it}^{k}$ directly translates into dispersion in capital costs, thereby contributing to misallocation (Hsieh and Klenow, 2009).

Excess dispersion in the cost to capital. Table 2 shows that the variance in the cost of capital, measured after demeaning for sector-year fixed effects, following standard practice. We find that in the base line sample, the variance is 0.46. Assuming a zero adjusted capital wedge reduces it to 0.13, about 72% of its original value, indicating that capital wedges may have sizable effects on aggregate productivity. In the subsample of firms with available proxies for frictions (as used in Section 4.3.3), the variance is 0.36 reduction is smaller but still

Table 2: Excess Dispersion in the Cost of Capital

	$\mathbb{V}(\mathcal{C}_{it}^k)$	$\mathbb{V}(\mathcal{C}_{it}^k \mid \boldsymbol{\tau}_{it}^{\text{adjusted}} = 0)$	$\mathbb{V}(\mathcal{C}_{it}^k \mid \widetilde{g(\Upsilon_{it})} = 0)$
Full Sample	0.46	0.13	–
Sub Sample	0.36	0.10	0.31

Note. Table 2 presents the variance in the cost of capital under various scenarios. $\mathbb{V}(\mathcal{C}_{it}^k)$ is variance in the measured cost of capital. $\mathbb{V}(\mathcal{C}_{it}^k \mid \boldsymbol{\tau}_{it}^{\text{adjusted}} = 0)$ is the variance in the cost of capital when assuming $\boldsymbol{\tau}_{it}^{\text{adjusted}} = 0$. $\mathbb{V}(\mathcal{C}_{it}^k \mid \widetilde{g(\Upsilon_{it})} = 0)$ is the variance when setting the predict part of the capital wedge from regression (30) in Section 4.3.3 to zero. The full sample represents the sample used throughout the paper, while the subsample refers to those firms with available predictors used to perform regression (30) in Section 4.3.3.

quantitative meaningful: setting the predicted wedge $\widetilde{\boldsymbol{\tau}_{it}^{\text{adjusted}}}$ from equation (30) to zero (i.e., $\widetilde{g(\Upsilon_{it})} = 0$) lowers the variance by 14%.[9]

Aggregate productivity. To infer aggregate productivity losses for the U.S. from the observed excess dispersion in the cost of capital, we impose additional structure on the representative household's demand as compared to the framework in Section 3. Specifically, we assume a standard CES demand aggregator given by:

$$\mathcal{D}(\{c_i\}) = \left(\sum_{i=1}^{M} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{32}$$

where σ denotes the elasticity of substitution across products. On the firm side, we assume the same production function used throughout the empirical analysis, given by:

$$q_i = z_i k_i^{\alpha} \ell_i^{1-\alpha}. \tag{33}$$

Moreover, firms face a heterogeneous cost of capital $\mathcal{C}_i \equiv \mathcal{R}_i^k + \delta_i$, as in Section 3. Given that the CES demand structure in equation (32) implies constant, homogeneous markups, to maintain consistency with the original framework and the empirical analysis, we introduce a firm-specific output tax, $(1 + \tau_i^y)$, which is observationally equivalent to allowing for heterogeneous markups across firms.

Finally, we can compute the total factor revenue productivity revenue ($TFPR$) that sum-

[9] The lower overall variance in this subsample reflects its composition, which includes primarily larger firms that are less constrained and likely closer to their optimal size.

marizes the distortions faced by the firms as,

$$TFPR_i = \underbrace{\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}}_{\widetilde{\alpha}} \underbrace{\frac{\sigma}{\sigma-1}\frac{1}{(1-\tau_i^y)}}_{\mu_i} (\mathcal{C}_i)^{\alpha}(p^{\ell})^{1-\alpha} = \widetilde{\alpha}\mu_i (\mathcal{C}_i)^{\alpha}(p^{\ell})^{1-\alpha} \tag{34}$$

In our context, there are two sources of dispersion in $TFPR$: heterogeneous markups across firms μ and heterogeneous costs of capital $\mathcal{C}$. In the absence of such distortions, $TFPR$ would be equalized across firms. Therefore, cross-firm variation in the cost of capital leads to deviations from this benchmark and reduces aggregate productivity.

Assuming the joint log-normality of $\{z_i, \tau_i^y, \mathcal{C}_i\}$, aggregate productivity losses coming from changes in dispersion in the cost of capital can be summarized as

$$\Delta \log TFP \approx -\left(\frac{\sigma\alpha^2}{2} + \frac{\alpha(1-\alpha)}{2}\right)\Delta\mathbb{V}(\log \mathcal{C}). \tag{35}$$

which can be used to quantify the aggregate productivity loss resulting from excess dispersion in the perceived cost of capital.[10]

The loss is pinned down by changes in the variance in the cost of capital that are equal to

$$\Delta\mathbb{V}(\log \mathcal{C}) = \mathbb{V}(\log \mathcal{C}) \times \gamma_{\text{excess}} \tag{36}$$

where γ_{excess} is the excess dispersion in the cost of capital measure (in percent) in the Table 2. γ_{excess} is between -0.72 and -0.13, with the former calculated as the counterfactual variance obtained by shutting down wedges (log(0.13)-log(0.46) from Table 2), while the latter is recovered as the counterfactual decrease in the variance from shutting down only the predicted part of the wedges (log(0.31)-log(0.36) from Table 2). We set the elasticity of substitution between products in a sector, σ, to 3 in our baseline calibration, in line with Broda and Weinstein (2006). We set α equal to 0.27, which is the average estimates from our empirical analysis.

The baseline estimate of the aggregate productivity losses, reported in Table 3, is 10.55%, based on the excess dispersion in the adjusted capital wedge from Table 2. Using the dispersion measure based on proxies for firm-level frictions, the estimated loss is 1.88%. When the

[10]The aggregate loss formula is an approximation, as it omits the covariance term $-\sigma\alpha\Delta\mathbb{C}(\log\mathcal{C}, \log\tau^y)$. This is a simplification to keep everything else constant and isolate the pure effect of changes in variance in cost of capital. We also check the covariance of $\mathcal{C}$ and τ^y under different counterfactual scenarios, which we found to moderately decline. Thus, our results can be considered as the lower bound of the overall effect.

elasticity of substitution σ is set to 4, the loss rises to 12.1%, and to 13.06% when σ equals 5. Higher values of σ imply greater substitutability across firms, increasing the scope for reallocation from high-τ to low-τ firms, and thus amplifying the gains from removing such distortions. These results suggest that excess dispersion in the cost of capital, driven by capital wedges, may have quantitatively substantial negative implication for aggregate productivity.

Table 3: Aggregate Productivity Losses from Excess Dispersion in the Cost of Capital

	$\gamma_{\text{excess}} = 72\%$	$\gamma_{\text{excess}} = 13\%$
TFP Loss (baseline, $\sigma = 3$)	10.55%	1.88%
TFP Loss ($\sigma = 4$)	12.06%	2.45%
TFP Loss ($\sigma = 5$)	13.06%	2.71%

7 Conclusion & Discussion

This paper develops a closed-form dynamic general equilibrium framework for disaggregated economies that decomposes the return on capital into firm-level contributions from markups, risk premia, and capital-related frictions. Although we confirm the important role of profits, a key novel finding is that, contrary to much of the existing literature emphasizing risk, rising capital frictions—driven by the increasing intensity of intangible capital among newer cohorts—have been the primary force preventing a decline in the return on capital.

Our findings carry important implications. First, after accounting for profits and measurement error, we find that the return on capital has declined—though not sufficiently to converge to the risk-free rate—and remains above per capita output growth. Moreover, capital frictions that have prevented a further decline in the aggregate return on capital may have reduced aggregate productivity by an estimated 2–13 percent.

This paper raises several challenging, albeit important, questions for future research. First, it identifies the proximate causes of the divergence between the return on capital and the risk-free rate, emphasizing the roles of markups, risk premia, and capital frictions. Although we do not address the fundamental causes of these components, future research aiming to do so must be guided by—and consistent with—the findings we highlight. Second, although we applied this framework to the divergence between the return on capital and the risk-free rate within the U.S., a promising extension is to explore how markups, risk premia, and frictions contribute to cross-country differences in capital returns.

References

Ackerberg, Daniel A and Jan De Loecker, "Production function identification under imperfect competition," Technical Report, Mimeo, University of Leuven 2021.

__ , Kevin Caves, and Garth Frazer, "Identification properties of recent production function estimators," *Econometrica*, 2015, *83* (6), 2411–2451.

Aum, Sangmin and Yongseok Shin, "Is Software Eating the World?," Technical Report, National Bureau of Economic Research 2024.

Avdis, Efstathios and Jessica A Wachter, "Maximum likelihood estimation of the equity premium," *Journal of Financial Economics*, 2017, *125* (3), 589–609.

Bai, Yan, Keyu Jin, and Dan Lu, "Misallocation under trade liberalization," *American Economic Review*, 2024, *114* (7), 1949–1985.

Baker, Scott R, Nicholas Bloom, and Steven J Davis, "Measuring economic policy uncertainty," *The quarterly journal of economics*, 2016, *131* (4), 1593–1636.

Baqaee, David Rezza and Emmanuel Farhi, "Productivity and misallocation in general equilibrium," *The Quarterly Journal of Economics*, 2020, *135* (1), 105–163.

Bauer, Michael D, Ben S Bernanke, and Eric Milstein, "Risk appetite and the risk-taking channel of monetary policy," *Journal of Economic Perspectives*, 2023, *37* (1), 77–100.

Bekaert, Geert and Marie Hoerova, "The VIX, the variance premium and stock market volatility," *Journal of econometrics*, 2014, *183* (2), 181–192.

__ , Eric C Engstrom, and Nancy R Xu, "The time variation in risk appetite and uncertainty," *Management Science*, 2022, *68* (6), 3975–4004.

Belo, Frederico, Vito D Gala, Juliana Salomao, and Maria Ana Vitorino, "Decomposing firm value," *Journal of Financial Economics*, 2022, *143* (2), 619–639.

Bils, Mark, Peter J Klenow, and Cian Ruane, "Misallocation or mismeasurement?," *Journal of Monetary Economics*, 2021, *124*, S39–S56.

Blundell, Richard and Stephen Bond, "Initial conditions and moment restrictions in dynamic panel data models," *Journal of econometrics*, 1998, *87* (1), 115–143.

Bøler, Esther Ann, Andreas Moxnes, and Karen Helene Ulltveit-Moe, "Strapped for cash: the role of financial constraints for innovating firms," 2023.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch, "Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data," *Journal of Monetary Economics*, 2021, *121*, 1–14.

Broda, Christian and David E Weinstein, "Globalization and the Gains from Variety," *The Quarterly journal of economics*, 2006, *121* (2), 541–585.

Brynjolfsson, Erik, Andrew McAfee, Michael Sorell, and Feng Zhu, "Scale without mass: business process replication and industry dynamics," *Harvard Business School Technology & Operations Mgt. Unit Research Paper*, 2008, (07-016).

Caballero, Ricardo J, Emmanuel Farhi, and Pierre-Olivier Gourinchas, "Rents, technical change, and risk premia accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares," *American Economic Review*, 2017, *107* (5), 614–620.

Caggese, Andrea and Ander Pérez-Orive, "How stimulative are low real interest rates for intangible capital?," *European Economic Review*, 2022, *142*, 103987.

Campbell, John Y and Samuel B Thompson, "Predicting excess stock returns out of sample: Can anything beat the historical average?," *The Review of Financial Studies*, 2008, *21* (4), 1509–1531.

Caselli, Francesco and James Feyrer, "The marginal product of capital," *The quarterly journal of economics*, 2007, *122* (2), 535–568.

Chiavari, Andrea and Sampreet Goraya, "The rise of intangible capital and the macroeconomic implications," *mimeo*, 2024.

Cloyne, James, Joseba Martinez, Haroon Mumtaz, and Paolo Surico, "Short-term tax cuts, long-term stimulus," Technical Report, National Bureau of Economic Research 2022.

Collard-Wexler, Allan and Jan De Loecker, "Productivity and capital measurement error," *NBER Working Paper*, 2021.

Crouzet, Nicolas and Janice Eberly, "Intangibles, investment, and efficiency," in "AEA Papers and Proceedings," Vol. 108 American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203 2018, pp. 426–431.

__ **and** __ , "Rents and intangible capital: A q+ framework," *The Journal of Finance*, 2023, *78* (4), 1873–1916.

__ **and Neil R Mehrotra**, "Small and large firms over the business cycle," *American Economic Review*, 2020, *110* (11), 3549–3601.

David, Joel M and Venky Venkateswaran, "The sources of capital misallocation," *American Economic Review*, 2019, *109* (7), 2531–2567.

__ **, Lukas Schmid, and David Zeke**, "Risk-adjusted capital allocation and misallocation," *Journal of Financial Economics*, 2022, *145* (3), 684–705.

Davis, Steven J, John Haltiwanger, Ron Jarmin, Javier Miranda, Christopher Foote, and Eva Nagypal, "Volatility and dispersion in business growth rates: Publicly traded versus privately held firms [with comments and discussion]," *NBER macroeconomics annual*, 2006, *21*, 107–179.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, "A model of monetary policy and risk premia," *The Journal of Finance*, 2018, *73* (1), 317–373.

Duarte, Fernando and Carlo Rosa, "The equity risk premium: a review of models," *Economic Policy Review*, 2015, (2), 39–57.

Eggertsson, Gauti B, Jacob A Robbins, and Ella Getz Wold, "Kaldor and Piketty's facts: The rise of monopoly power in the United States," *Journal of Monetary Economics*, 2021, *124*, S19–S38.

Ewens, Michael, Ryan H Peters, and Sean Wang, "Measuring intangible capital with market prices," *Management Science*, 2024.

Falato, Antonio, Dalida Kadyrzhanova, Jae Sim, and Roberto Steri, "Rising intangible capital, shrinking debt capacity, and the US corporate savings glut," *The Journal of Finance*, 2022, *77* (5), 2799–2852.

Fama, Eugene F and Kenneth R French, "Production of US SMB and HML in the Fama-French Data Library," *Chicago Booth Research Paper*, 2023, (23-22).

Farhi, Emmanuel and François Gourio, "Accounting for macro-finance trends: Market power, intangibles, and risk premia," Technical Report, National Bureau of Economic Research 2018.

Gagliardini, Patrick, Elisa Ossola, and Olivier Scaillet, "Time-varying risk premium in large cross-sectional equity data sets," *Econometrica*, 2016, *84* (3), 985–1046.

Gandhi, Amit, Salvador Navarro, and David A Rivers, "On the identification of gross output production functions," *Journal of Political Economy*, 2020, *128* (8), 2973–3016.

Gilchrist, Simon and Egon Zakrajšek, "Credit spreads and business cycle fluctuations," *American economic review*, 2012, *102* (4), 1692–1720.

Gomme, Paul, B Ravikumar, and Peter Rupert, "The return to capital and the business cycle," *Review of Economic Dynamics*, 2011, *14* (2), 262–278.

_ , _ , and _ , "Secular stagnation and returns on capital," *Economic Synopses*, 2015, (19).

Gormsen, Niels Joachim and Kilian Huber, "Corporate discount rates," Technical Report, National Bureau of Economic Research 2023.

Hall, Robert E, "The relation between price and marginal cost in US industry," *Journal of political Economy*, 1988, *96* (5), 921–947.

Hoberg, Gerard and Vojislav Maksimovic, "Redefining financial constraints: A text-based analysis," *The Review of Financial Studies*, 2015, *28* (5), 1312–1352.

Hou, Kewei, Chen Xue, and Lu Zhang, "Digesting anomalies: An investment approach," *The Review of Financial Studies*, 2015, *28* (3), 650–705.

Hsieh, Chang-Tai and Peter J Klenow, "Misallocation and manufacturing TFP in China and India," *The Quarterly journal of economics*, 2009, *124* (4), 1403–1448.

III, Donald E Bowen, Laurent Frésard, and Gerard Hoberg, "Rapidly evolving technologies and startup exits," *Management Science*, 2023, *69* (2), 940–967.

Jagannathan, Ravi, Ellen McGrattan, and Anna Scherbina, "The declining US equity premium," 2001.

Jordà, Òscar, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick, and Alan M Taylor, "The rate of return on everything, 1870–2015," *The quarterly journal of economics*, 2019, *134* (3), 1225–1298.

Jurado, Kyle, Sydney C Ludvigson, and Serena Ng, "Measuring uncertainty," *American Economic Review*, 2015, *105* (3), 1177–1216.

Koh, Dongya, Raül Santaeulàlia-Llopis, and Yu Zheng, "Labor share decline and intellectual property products capital," *Econometrica*, 2020, *88* (6), 2609–2628.

Lettau, Martin, Sydney C Ludvigson, and Jessica A Wachter, "The declining equity premium: What role does macroeconomic risk play?," *The Review of Financial Studies*, 2008, *21* (4), 1653–1687.

Linn, Matthew and Daniel Weagley, "Uncovering financial constraints," *Journal of Financial and Quantitative Analysis*, 2024, *59* (6), 2582–2617.

Lintner, John, "Security prices, risk, and maximal gains from diversification," *The journal of finance*, 1965, *20* (4), 587–615.

_ , "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *The Review of Economics and Statistics*, 1965, *47* (1), 13–37.

Loecker, Jan De and Frederic Warzynski, "Markups and firm-level export status," *American economic review*, 2012, *102* (6), 2437–2471.

_ , Jan Eeckhout, and Gabriel Unger, "The rise of market power and the macroeconomic implications," *The Quarterly Journal of Economics*, 2020, *135* (2), 561–644.

Ma, Yueran, Benjamin Pugsley, and Kaspar Zimmermann, "Superstar Firms through the Generations," Technical Report, Working Paper 2024.

Martin, Ian, "What is the Expected Return on the Market?," *The Quarterly Journal of Economics*, 2017, *132* (1), 367–433.

Marx, Magali, Benoît Mojon, and François R Velde, "Why have interest rates fallen far below the return on capital?," *Journal of Monetary Economics*, 2021, *124*, S57–S76.

Moreira, Sara, "Firm dynamics, persistent effects of entry conditions, and business cycles," *Persistent Effects of Entry Conditions, and Business Cycles (October 1, 2016)*, 2016.

Mossin, Jan, "Equilibrium in a capital asset market," *Econometrica: Journal of the econometric* *society*, 1966, pp. 768–783.

Peters, Ryan H and Lucian A Taylor, "Intangible capital and the investment-q relation," *Journal of Financial Economics*, 2017, *123* (2), 251–272.

Piketty, Thomas, *Capital in the twenty-first century*, Harvard University Press, 2014.

Rachel, Łukasz and Lawrence H Summers, "On secular stagnation in the industrialized world," Technical Report, National Bureau of Economic Research 2019.

Reis, Ricardo, "Which r-star, public bonds or private investment? Measurement and policy implications," *LSE manuscript*, 2022.

Ridder, Maarten De, Basile Grassi, and Giovanni Morzenti, "The Hitchhiker's Guide to Markup Estimation," 2024.

Sedláček, Petr and Vincent Sterk, "The growth potential of startups over the business cycle," *American Economic Review*, 2017, *107* (10), 3182–3210.

Sharpe, William F, "Capital asset prices: A theory of market equilibrium under conditions of risk," *The journal of finance*, 1964, *19* (3), 425–442.

Sterk, Vincent, Petr Sedláček, and Benjamin Pugsley, "The nature of firm growth," *American Economic Review*, 2021, *111* (2), 547–579.

Summers, Lawrence H, "US economic prospects: Secular stagnation, hysteresis, and the zero lower bound," *Business economics*, 2014, *49* (2), 65–73.

Syverson, Chad, "Markups and Markdowns," 2024.

Treynor, Jack L, *Toward a theory of market value of risky assets* 1962.

Zhang, Lichen, "Non-rival Intangibles and the Life-Cycle Dynamics of Superstar Firms," Technical Report, mimeo 2024.

The Divergence Between the Return of Capital and the Risk-Free Rate: A Micro-Anatomy

Andrea Chiavari, Julia Faltermeier, and Sampreet S. Goraya

Online Appendix

Contents

A Additional Evidence on the Return on Capital and Risk-Free Rate Divergence

Here, we explain how the return on capital is calculated in the main text, present its historical evolution in comparison to the risk-free rate, and demonstrate that the divergence between risk-free rate and return on cpaital remains robust across various measures of both return on capital and the risk-free rate.

Measurement of return of capital. To measure the aggregate return on capital in the national accounts, we follow the equation (1). For aggregate measures we follow Koh et al. (2020). In practice, our baseline measure of the return on capital is calculated as follows:

$$R^k = \frac{NOS - CE - PI - DEP}{K}, \tag{A.1}$$

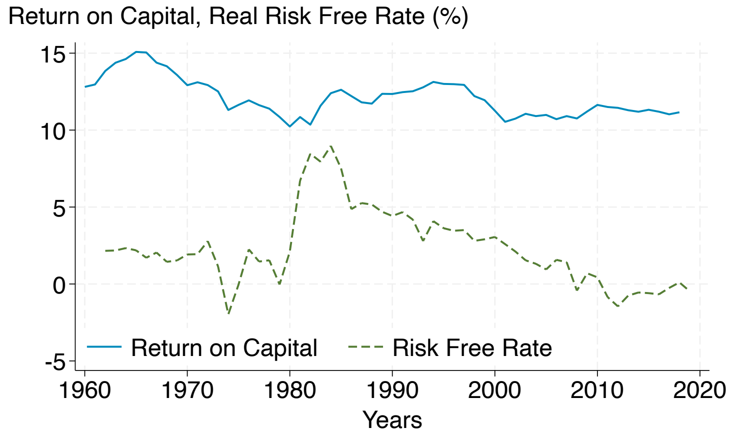
where NOS is the net operating surplus, CE represents compensation for employees (from NIPA Table 1.12), PI refers to proprietors' income (also from NIPA Table 1.12), DEP stands for depreciation (from the Fixed Assets Accounts Tables), and K represents the fixed assets as defined by the BEA. This includes all non-residential structures, equipment, and intellectual property products (IPP).

Historical evolution of return of capital versus the risk-free rate. Figure A.1 presents the evolution of the return on capital and the risk-free rate since the 1960s. We find that the return on capital remains relatively flat over these six decades, consistent with the findings of Gomme et al. (2011) and Reis (2022). In contrast, the risk-free rate follows an inverted U-shaped pattern, in line with Rachel and Summers (2019).

Additional measures of the return on capital and the risk-free rate. Figures A.2a and A.2b show the evolution of alternative measures of the return on capital and the risk-free rate since the 1980s.

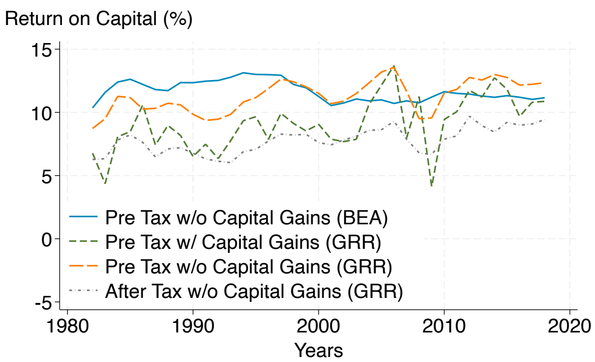
Figures A.2a illustrates the evolution of several alternative measures of the return on capital from Gomme et al. (2011). These measures focus on the business sector, thus excluding housing, and explicitly account for taxation and capital gains. While the inclusion of taxa-

Figure A.1: Historical Evolution of the Return on Capital and Risk-Free Rate

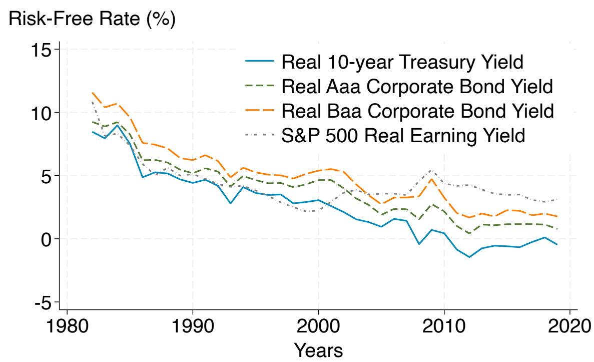


Note. Figure A.1 shows the evolution of the return on capital and the risk-free rate, measured in percent, since 1960. The return on capital is measured using equation (2) with BEA data. The risk-free rate is the market yield on U.S. Treasury securities with a 10-year constant maturity net of expected inflation from Michigan.

Figure A.2: Alternative Measures of Return on Capital and Risk-Free Rate Over Time



(a) Return on Capital



(b) Risk-Free Rate

Note. Figure A.2a presents the evolution of alternative measures of the return on business capital from Gomme et al. (2011), both before and after tax, and with and without capital gains. Figure A.2b displays alternative measures of the risk-free rate, as reported in Rachel and Summers (2019).

tion and capital gains has only a modest impact on the level of the return on capital, it does not affect its remarkable stability over time. This robustness to various adjustments is also documented in Reis (2022).

Figure A.2b presents the evolution of several alternative measures of the risk-free rate, similar to those used in Rachel and Summers (2019). Specifically, it shows the real Aaa Corporate Bond yield, the real Bbb Corporate Bond yield, and the S&P 500 Real Earnings yield, all of which exhibit a notably similar decline over time.

B Further Details on the Microfounded Structural Decomposition

B.1 Additional Microfoundations for the Firms' Problem

In this section, we present alternative microfoundations for the capital wedge $\boldsymbol{\tau}$, as used in the main text. Specifically, we demonstrate that these wedges can be interpreted as financial frictions, adjustment costs, or measurement error.

Financial frictions. Assuming the presence of time-varying firm i specific capitalconstraints κ_{it}, capturing the flexible presence of financial friction constraining capital expansions of the firm, then the Lagrangian objective function associated with the firm's cost minimization problem can be expressed as:

$$\begin{aligned}\mathcal{L}(\ell_{it}, k_{it}, \boldsymbol{\tau}_{it}, \xi_{it}) = p^{\ell}_{it}\ell_{it} + \sum_{j\in\{T,I\}} \left(r_t + \boldsymbol{\zeta}_{it} + \delta_{it}\right) p_t k_{it} + \boldsymbol{c}_{it} \\ - \boldsymbol{\tau}_{it}\left(\kappa_{it} - p_t k_{it}\right) - \xi_{it}\left(q(\cdot) - q_{it}\right),\end{aligned} \tag{B.2}$$

where all variables are defined as in the main text, except that $\boldsymbol{\tau}_{it}$ now represents the Lagrange multiplier associated with the constraint. The first-order conditions for both types of capital, along with the complementary slackness condition, are given by:

$$r_t + \boldsymbol{\zeta}_{it} + \delta_{it} + \boldsymbol{\tau}_{it} = \frac{\xi_{it}}{p_t}\frac{\partial q(\cdot)}{\partial k_{it}} \tag{B.3}$$

$$\left(\kappa_{it} - p_t k_{it}\right) \geq 0 \tag{B.4}$$

$$\boldsymbol{\tau}_{it}\left(p_t k_{it} - \kappa_{it}\right) = 0. \tag{B.5}$$

Note that equation (B.3) is isomorphic to equation (6) in the main text, as the right-hand side is exactly equal to $\widetilde{MRPK}_{it}$.

Adjustment costs. Assuming that firms face capital-specific adjustment frictions, $\boldsymbol{\tau}(k_{it})$, the Lagrangian objective function for the firm's cost minimization problem can be expressed as:

$$\mathcal{L}(\ell_{it}, k_{it}, \xi_{it}) = p^{\ell}_{it}\ell_{it} + (r_t + \boldsymbol{\zeta}_{it} + \delta_{it})\, p_t k_{it} - p_t \boldsymbol{\tau}(k_{it}) + \boldsymbol{c}_{it} - \xi_{it}\,(q(\cdot) - q_{it})\,. \tag{B.6}$$

The first-order conditions for both types of capital are given by:

$$r_t + \boldsymbol{\zeta}_{it} + \delta_{it} + \boldsymbol{\tau}_{it} = \frac{\xi_{it}}{p_t}\frac{\partial q(\cdot)}{\partial k_{it}}, \tag{B.7}$$

where $\boldsymbol{\tau}_{it} \equiv \partial \boldsymbol{\tau}(k_{it})/\partial k_{it}$. Again, note that equation (B.7) is isomorphic to equations in the main text, as the right-hand side is exactly equal to $\widetilde{MRPK^{o}}_{it}$.

B.2 Proof of Proposition 1

The firm problem is

$$\mathcal{L}(\{\ell_{it}\}, \{k_{it}\}, \xi_{it}) = \sum_{\ell^n} p_t^{\ell^n}\ell^n_{it} + \sum_{k^o}\left(r_t + \boldsymbol{\zeta}^{k^o}_{it} + \delta^{k^o} + \boldsymbol{\tau}^{k^o}_{it}\right) p_t^{k^o} k^o_{it} + \boldsymbol{c}_{it} - \xi_{it}\,(q(\cdot) - q_{it})\,, \tag{B.8}$$

First order condition with respect to a variable input ℓ^n_{it} is

$$\begin{gathered}
\frac{\partial \mathcal{L}}{\partial \ell^n_{it}} = p_t^{\ell^n} - \xi_{it}\frac{\partial q_{it}}{\partial \ell^n_{it}} = 0 \quad \Longrightarrow \quad p_t^{\ell^n} = \xi_{it}\frac{\partial q_{it}}{\partial \ell^n_{it}} \\
p_t^{\ell^n} = \xi_{it}\frac{q_{it}}{\ell^n_{it}}\frac{\partial \log q(\cdot)}{\partial \log \ell^n_{it}} = \xi_{it}\mathcal{E}_{\ell^n}\frac{q_{it}}{\ell^n_{it}} \\
1 = \frac{\xi_{it}}{p_{it}}\mathcal{E}_{\ell^n}\frac{p_{it}q_{it}}{p_t^{\ell^n}\ell^n_{it}} \quad \Longrightarrow \quad \boldsymbol{\mu_{it}} = \mathcal{E}_{\ell^n}\frac{p_{it}q_{it}}{p_t^{\ell^n}\ell^n_{it}}
\end{gathered} \tag{B.9}$$

First order condition with respect to a capital input k^n_{it} is

$$\begin{gathered}
\left(r_t + \boldsymbol{\zeta}^{k^o}_{it} + \delta^{k^o} + \boldsymbol{\tau}^{k^o}_{it}\right) p_t^{k^o} = \xi_{it}\frac{\partial q_{it}}{\partial k^o_{it}} \\
r_t + \boldsymbol{\zeta}^{k^o}_{it} + \delta^{k^o} + \boldsymbol{\tau}^{k^o}_{it} = \mathcal{E}_{k^o}\frac{1}{\boldsymbol{\mu_{it}}}\frac{p_{it}q_{it}}{p_t^{\ell^n}k^o_{it}}
\end{gathered} \tag{B.10}$$

B.3 Proof of Proposition 2

A representative household solves

$$\mathcal{L} = \max_{C_t, B_{t+1}, \{c_{it}, k_{it+1}\}\forall i} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t; Z_t)$$

$$+ \lambda_t \left(\sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \left(\underbrace{(r_t + \zeta_{it}^{k^o} + \delta^{k^o})}_{MRPK_{it}^o} p_t^{k^o} k_{it}^o - p_{t+1}^{k^o} \left(k_{it+1}^o - (1-\delta^{k^o}) k_t^o\right) \right) \right) + (1+r_t)B_t + \Pi_t$$

$$- P_t C_t - B_{t+1} \tag{B.11}$$

The first order conditions are

$$C_t :\Rightarrow \quad U_C(C_t; Z_t) = \lambda_t P_t$$

$$B_{t+1} :\Rightarrow -\lambda_t + \beta E_t[\lambda_{t+1}(1+r_{t+1})] = 0 \tag{B.12}$$

$$k_{it+1}^o :\Rightarrow -\lambda_t E_t p_{t+1}^{k^o} + \beta E_t\big[\lambda_{t+1}\big((r_{t+1} + \zeta_{it+1}^{k^o} + \delta^{k^o}) p_{t+1}^{k^o} + (1-\delta^{k^o}) p_{t+2}^{k^o}\big)\big] = 0$$

Define the nominal SDF

$$M_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{U_C(C_{t+1}; Z_{t+1})}{U_C(C_t; Z_t)} \frac{P_t}{P_{t+1}}. \tag{B.13}$$

Insert in the B_{t+1} FOC:

$$1 = \beta\, E_t\left[\frac{U_C(C_{t+1}; Z_{t+1})}{U_C(C_t; Z_t)} \frac{P_t}{P_{t+1}} (1 + r_{t+1})\right] \iff 1 = E_t[M_{t+1}(1+r_{t+1})]\,. \tag{B.14}$$

Insert the SDF into the k_{it+1} FOC:

$$\begin{aligned} 1 &= \beta\, E_t\left[\frac{U_C(C_{t+1}; Z_{t+1})}{U_C(C_t; Z_t)} \frac{P_t}{P_{t+1}} \Big((r_{t+1} + \zeta_{it+1}^{k^o} + \delta^{k^o}) + (1-\delta^{k^o}) \frac{p_{t+2}^{k}}{p_{t+1}^{k^o}}\Big)\right] \\ 1 &= \beta\, E_t\left[M_t\Big(MRPK_{it+1} + (1-\delta^{k^o}) \frac{p_{t+2}^{k^o}}{p_{t+1}^{k^o}}\Big)\right] \end{aligned} \tag{B.15}$$

From equation (10) it is easy to get:

$$1 + r_{t-1} = \mathbb{E}_{t-1}\left[MRPK_{it} + \kappa_{it-1}(1-\delta^j)\frac{p^{k^o}_{t+1}}{p^{k^o}_t}\right] + \frac{\mathbb{C}_{t-1}\left(M_t, MRPK_{it}\right)}{\mathbb{E}_{t-1}\left[M_t\right]}, \tag{B.16}$$

since $\mathbb{E}_{t-1}\left[M_t\right] = (1+r_{t-1})^{-1}$ and $\mathbb{C}_{t-1}\left(M_t, (1-\delta^{k^o})\frac{p^{k^o}_{t+1}}{p^{k^o}_t}\right) = 0$. Substituting into equation (B.16) equation (7), replacing expectations with an expectational error ε_{it}, and rearranging the terms yields,

$$\widetilde{MRPK}^o_{it} - r_{t-1} = \boldsymbol{\tau}^{k^o}_{it} + 1 - (1-\delta^{k^o})\frac{p^{k^o}_{t+1}}{p^{k^o}_t} - \frac{\mathbb{C}_{t-1}\left(M_t, MRPK^o_{it}\right)}{\mathbb{E}_{t-1}\left[M_t\right]} + \varepsilon_{it} \tag{B.17}$$

B.4 Proof of Proposition 3

We start with the definition of the return on capital and add and subtract risk premia, risk-free rate and capital frictions from the right hand side and we get,

$$\begin{aligned} R^k_{it} &\equiv \frac{p_{it}q_{it} - \boldsymbol{c}_{it} - \sum_{k^o\in\{k\}}\delta^{k^o}p^{k^o}_t k^o_{it} - \sum_{\ell^n\in\{\ell\}}p^{\ell^n}_t \ell^n_{it}}{\sum_{k^o\in\{k\}}p^{k^o}_t k^o_{it}} \\ &= \frac{p_{it}q_{it} - \boldsymbol{c}_{it} - \sum_{k^o\in\{k\}}(r_t + \boldsymbol{\zeta}^{k^o}_{it} + \delta^{k^o} + \boldsymbol{\tau}^{k^o}_{it})p^{k^o}_t k^o_{it} - \sum_{\ell^n\in\{\ell\}}p^{\ell^n}_t \ell^n_{it}}{\sum_{k^o\in\{k\}}p^{k^o}_t k^o_{it}} \\ &+ \sum_{k^o\in\{k\}}\frac{r_t p^{k^o}_t k^o_{it}}{\sum_{k^o\in\{k\}}p^{k^o}_t k^o_{it}} + \sum_{k^o\in\{k\}}\frac{\boldsymbol{\tau}^{k^o}_{it}p^{k^o}_t k^o_{it}}{\sum_{k^o\in\{k\}}p^{k^o}_t k^o_{it}} + \sum_{k^o\in\{k\}}\frac{\boldsymbol{\zeta}^{k^o}_{it}p^{k^o}_t k^o_{it}}{\sum_{k^o\in\{k\}}p^{k^o}_t k^o_{it}} \end{aligned} \tag{B.18}$$

We can use the first order conditions proved in Proposition 1, and replace in the Equation (B.18)

$$\begin{aligned} R^k_{it} = & \frac{p_{it}q_{it} - \boldsymbol{c}_{it} - \sum_{k^o\in\{k\}}\mathcal{E}_{k^o}\frac{1}{\boldsymbol{\mu_{it}}}p_{it}q_{it} - \sum_{\ell^n\in\{\ell\}}\mathcal{E}_{\ell^n}\frac{1}{\boldsymbol{\mu_{it}}}p_{it}q_{it}}{\sum_{k^o\in\{k\}}p^{k^o}_t k^o_{it}} \\ & + r_t + \sum_{k^o\in\{k\}}\frac{\boldsymbol{\tau}^{k^o}_{it}p^{k^o}_t k^o_{it}}{\sum_{k^o\in\{k\}}p^{k^o}_t k^o_{it}} + \sum_{k^o\in\{k\}}\frac{\boldsymbol{\zeta}^{k^o}_{it}p^{k^o}_t k^o_{it}}{\sum_{k^o\in\{k\}}p^{k^o}_t k^o_{it}} \end{aligned} \tag{B.19}$$

By realigning terms, we can have

$$
\begin{aligned}
R_{it}^{k} &= \frac{p_{it}q_{it} - p_{it}q_{it}\left(\sum_{k^o\in\{k\}} \mathcal{E}_{k^o}\frac{1}{\boldsymbol{\mu_{it}}} + \sum_{\ell^n\in\{\ell\}} \mathcal{E}_{\ell^n}\frac{1}{\boldsymbol{\mu_{it}}}\right)}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}} - \frac{\boldsymbol{c}_{it}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}} \\
&+ \sum_{k^o\in\{k\}} \frac{\boldsymbol{\tau}_{it}^{k^o} p_t^{k^o} k_{it}^{o}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}} + \sum_{k^o\in\{k\}} \frac{\boldsymbol{\zeta}_{it}^{k^o} p_t^{k^o} k_{it}^{o}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}} \\
&= \underbrace{\frac{p_{it}q_{it}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}}\left(1 - \frac{\sum_{k^o\in\{k\}} \mathcal{E}_{k^o} + \sum_{\ell^n\in\{\ell\}} \mathcal{E}_{\ell^n}}{\boldsymbol{\mu_{it}}}\right) - \frac{\boldsymbol{c}_{it}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}}}_{\pi_{it}} \\
&+ \underbrace{r_t + \sum_{k^o\in\{k\}} \frac{\boldsymbol{\tau}_{it}^{k^o} p_t^{k^o} k_{it}^{o}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}} + \sum_{k^o\in\{k\}} \frac{\boldsymbol{\zeta}_{it}^{k^o} p_t^{k^o} k_{it}^{o}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}}}_{\mathcal{R}_{it}^{k}}
\end{aligned}
\tag{B.20}
$$

Therefore, the profits are

$$
\pi_{it} \equiv \frac{p_{it}q_{it}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}}\left(1 - \frac{\sum_{k^o\in\{k\}} \mathcal{E}_{k^o} + \sum_{\ell^n\in\{\ell\}} \mathcal{E}_{\ell^n}}{\boldsymbol{\mu}_{it}}\right) - \frac{\boldsymbol{c}_{it}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}^{o}}, \tag{B.21}
$$

B.5 Proof of Proposition 4

We start from the definition of aggregate return on capital,

$$
R_t^k = \sum_{i\in\mathcal{I}} \omega_{it}\left(\frac{p_{it}q_{it} - \boldsymbol{c}_{it} - \sum_{k^o\in\{k\}} \delta^{k^o} p_t^{k^o} k_{it}^{o} - \sum_{\ell^n\in\{\ell^n\}} p_t^{\ell} \ell_{it}}{\sum_{k^o\in\{k\}} p_t^{k^o} k_{it}}\right) = \sum_{i\in\mathcal{I}} \omega_{it} R_{it}^{k}; \tag{B.22}
$$

Building on the Proposition 4, we can get

$$
R_t^k = \mathcal{R}_t^k + \Pi_t = \sum_{i\in\mathcal{I}} \omega_{it}\left(r_t + \sum_{k^o\in\{k\}} \kappa_{it}^{k^o}\boldsymbol{\zeta}_{it}^{k^o} + \sum_{k^o\in\{k\}} \kappa_{it}^{k^o}\boldsymbol{\tau}_{it}^{k^o} + \pi_{it}\right); \tag{B.23}
$$

where $\mathcal{R}_t^k \equiv \sum_i \omega_{it}\mathcal{R}_{it}^k$ is the *true* aggregate return on capital and $\Pi_t \equiv \sum_i \omega_{it}\pi_{it}$ is the aggregate profit rate. Equation (B.23) demonstrates how the measured aggregate return on capital can be represented as a capital-weighted average of the various firm-level narratives identified in the literature.

Rearranging equation (B.23), we write the difference between the return on capital and the risk-free rate as follows:

$$R_t^k - r_t = \underbrace{\sum_{i\in\mathcal{I}} \omega_{it}\left(\mathcal{R}_{it}^k - r_t\right)}_{\text{True return on capital}} + \underbrace{\sum_{i\in\mathcal{I}} \omega_{it}\pi_{it}}_{\text{Profits}}, \tag{B.24}$$

$$= \underbrace{\sum_{i\in\mathcal{I}}\sum_{k^o\in\{k\}} \omega_{it}\kappa_{it}^{k^o}\zeta_{it}^{k^o}}_{\text{Risk}} + \underbrace{\sum_{i\in\mathcal{I}}\sum_{k^o\in\{k\}} \omega_{it}\kappa_{it}^{k^o}\boldsymbol{\tau}_{it}^{k^o}}_{\text{Capital-Wedges}} + \underbrace{\sum_{i\in\mathcal{I}} \omega_{it}\pi_{it}}_{\text{Profits}}. \tag{B.25}$$

B.6 Proof of Proposition 5

Let us define within-sector firm's capital shares $\hat{\omega}_{it} = \frac{\omega_{it}}{\sum_{i\in s}\omega_{it}}$

$$\sum_{i\in\mathcal{I}}\sum_{k^o\in\{k\}} \omega_{it}\kappa_{it}^{k^o}\zeta_{it}^{k^o} = \sum_{s}\sum_{k^o\in\{k\}} \omega_{st}\zeta_{st}^{k^o}, \quad \text{where} \quad \zeta_{st}^{k^o} \equiv \sum_{i\in s}\hat{\omega}_{it}\kappa_{it}^{k^o}\zeta_{it}^{k^o} \quad \text{and} \quad \omega_{st} \equiv \frac{\sum_{i\in s}\omega_{it}}{\sum_i \omega_{it}}. \tag{B.26}$$

Now, looking at the change in aggregate risk premia is and adding and subtracting few terms, we get,

$$\Delta\left(\sum_{s}\sum_{k^o\in\{k\}} \omega_{st}\zeta_{st}^{k^o}\right) = \sum_{s}\sum_{k^o\in\{k\}} \omega_{st}\zeta_{st}^{k^o} - \sum_{s}\sum_{k^o\in\{k\}} \omega_{st-1}\zeta_{st-1}^{k^o}$$

$$= \left[\sum_{s}\sum_{k^o\in\{k\}} \omega_{st-1}\,\zeta_{st}^{k^o} - \sum_{s}\sum_{k^o\in\{k\}} \omega_{st-1}\,\zeta_{st-1}^{k^o}\right] + \left[\sum_{s}\sum_{k^o\in\{k\}} \omega_{st}\,\zeta_{st-1}^{k^o} - \sum_{s}\sum_{k^o\in\{k\}} \omega_{st-1}\,\zeta_{st-1}^{k^o}\right]$$

$$` + \left[\sum_{s}\sum_{k^o\in\{k\}} \omega_{st}\,\zeta_{st}^{k^o} - \sum_{s}\sum_{k^o\in\{k\}} \omega_{st-1}\,\zeta_{st}^{k^o}\right] + \left[\sum_{s}\sum_{k^o\in\{k\}} \omega_{st-1}\,\zeta_{st-1}^{k^o} - \sum_{s}\sum_{k^o\in\{k\}} \omega_{st}\,\zeta_{st-1}^{k^o}\right]$$

$$= \sum_{s}\sum_{k^o\in\{k\}} \omega_{s,t-1}\Delta\zeta_{st}^{k^o} + \sum_{s}\sum_{k^o\in\{k\}} \Delta\omega_{s,t}\zeta_{st-1}^{k^o} + \sum_{s}\sum_{k^o\in\{k\}} \Delta\omega_{s,t}\Delta\zeta_{st}^{k^o} \tag{B.27}$$

In the same way, we can derive changes in the aggregate profits and aggregate capital frcitions, combining them, we get,

$$\Delta(R_t^k - r_t) = \Delta\left(\sum_s \sum_{k^o\in\{k\}} \omega_{st}\boldsymbol{\zeta}_{st}^{k^o}\right) + \Delta\left(\sum_s \sum_{k^o\in\{k\}} \omega_{st}\boldsymbol{\tau}_{st}^{k^o}\right) + \Delta\left(\sum_s \sum_{k^o\in\{k\}} \omega_{st}\pi_{st}\right)$$

$$= \sum_s \sum_{k^o\in\{k\}} \omega_{s,t-1}\Delta\boldsymbol{\zeta}_{st}^{k^o} + \sum_s \sum_{k^o\in\{k\}} \Delta\omega_{s,t}\boldsymbol{\zeta}_{st-1}^{k^o} + \sum_s \sum_{k^o\in\{k\}} \Delta\omega_{s,t}\Delta\boldsymbol{\zeta}_{st}^{k^o}$$

$$+ \sum_s \sum_{k^o\in\{k\}} \omega_{s,t-1}\Delta\boldsymbol{\tau}_{st}^{k^o} + \sum_s \sum_{k^o\in\{k\}} \Delta\omega_{s,t}\boldsymbol{\tau}_{st-1}^{k^o} + \sum_s \sum_{k^o\in\{k\}} \Delta\omega_{s,t}\Delta\boldsymbol{\tau}_{st}^{k^o}$$

$$+ \sum_s \sum_{k^o\in\{k\}} \omega_{s,t-1}\Delta\pi_{st}^{k^o} + \sum_s \sum_{k^o\in\{k\}} \Delta\omega_{s,t}\pi_{st-1}^{k^o} + \sum_s \sum_{k^o\in\{k\}} \Delta\omega_{s,t}\Delta\pi_{st}^{k^o}, \tag{B.28}$$

B.7 Proof of Proposition 6

We start providing the decomposition for the aggregate risk permia,

$$\sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \omega_{it}\kappa_{it}^{k^o}\boldsymbol{\zeta}_{it}^{k^o} = \sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \left(\overline{\boldsymbol{\zeta}_t} + \boldsymbol{\zeta}_{it}^{k^o} - \overline{\boldsymbol{\zeta}_t}\right)\left(\overline{\omega_t\kappa_t} + \omega_{it}\kappa_{it}^{k^o} - \overline{\omega_t\kappa_t}\right)$$

$$= \overline{\boldsymbol{\zeta}_t} + \sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \left(\boldsymbol{\zeta}_{it}^{k^o} - \overline{\boldsymbol{\zeta}_t}\right)\left(\omega_{it}\kappa_{it}^{k^o} - \overline{\omega_t\kappa_t}\right) \tag{B.29}$$

where $\overline{\boldsymbol{\zeta}_t}$ is changes in average risk permia, While $\overline{\omega_t\kappa_t}$ is the average share of a specific capital type across the firms. The changes over time can be written as

$$\Delta \sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \omega_{it}\kappa_{it}^{k^o}\boldsymbol{\zeta}_{it}^{k^o} = \Delta\overline{\boldsymbol{\zeta}_t} + \Delta \sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \left(\boldsymbol{\zeta}_{it}^{k^o} - \overline{\boldsymbol{\zeta}_t}\right)\left(\omega_{it}\kappa_{it}^{k^o} - \overline{\omega_t\kappa_t}\right) \tag{B.30}$$

Similarly, one can decompose profits and capital frictions, then we add them together to have,

$$\Delta(R_t^k - r_t) = \Delta\overline{\boldsymbol{\zeta}_t} + \Delta \sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \left(\boldsymbol{\zeta}_{it}^{k^o} - \overline{\boldsymbol{\zeta}_t}\right)\left(\omega_{it}\kappa_{it}^{k^o} - \overline{\omega_t\kappa_t}\right)$$

$$+ \Delta\overline{\boldsymbol{\tau}_t} + \Delta \sum_{i\in\mathcal{I}} \sum_{k^o\in\{k\}} \left(\boldsymbol{\tau}_{it}^{k^o} - \overline{\boldsymbol{\tau}_t}\right)\left(\omega_{it}\kappa_{it}^{k^o} - \overline{\omega_t\kappa_t}\right) \tag{B.31}$$

$$+ \Delta\overline{\pi_t} + \Delta \sum_{i\in\mathcal{I}} (\pi_{it} - \pi_t)(\omega_{it} - \overline{\omega_t}).$$

C Further Details on Data, Variable Construction, and Measurement

C.1 Data Cleaning and Summary Statistics

Here, we explain the data cleaning process. For data quality purposes, we interpret values for `sale`, k_{it}, k_{it}^I, `cogs`, or `xsga` as errors if they are zero, negative, or missing, and we exclude those observations. If `xrd`, `intano`, or `am` are negative or missing, we treat them as zeros. To make all variables real, we deflate them using the GDP deflator. Finally, we winsorize `sale`, k_{it}, k_{it}^I, `cogs`, and `xsga` at the 0.5 percent level. Table C.1 provides summary statistics for these variables.

Table C.1: Summary Statistics

	Sale	Cost of Goods Sold	Tangible Capital Stock	Intangible Capital Stock	Selling General & Administrative
Mean	1088,446	716,663	681,002	344,926	128,177
p25	24,905	13,199	6,812	7,985	4,291
p50	123,044	69,643	35,971	33,523	16,037
p75	586,797	350,030	218,133	152,703	68,144
No. Obs.	161,317	161,317	161,317	161,317	161,317

Note. Summary statistics of cleaned Compustat dataset between 1982 and 2019. All variables are in thousands of U.S.$.

C.2 Additional Production Functions and Markups Measurements

Production Function Estimation with no First Stage. Relative to the benchmark production function, we follow production function estimation using Blundell and Bond (1998), which does not include the first stage.

Production Function Estimation with Measurement Error. Following the methodology developed by Collard-Wexler and De Loecker (2021), we allow classical measurement error in the capital stock, and restimate the production function elasticities.

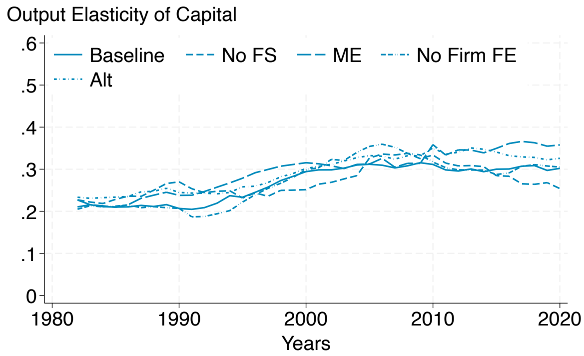
Production Function Estimation a la Ackerberg and De Loecker (2021). Here, we do not allow for firm fixed effects; this methodology closely follows the standard control function approach.

Production Function Estimation with Alternative Measure of Intangible Capital. Here, we estimate the intangible capital stock by assuming the first-period stock as the ratio of the first year's investment in intangibles over the depreciation rate.

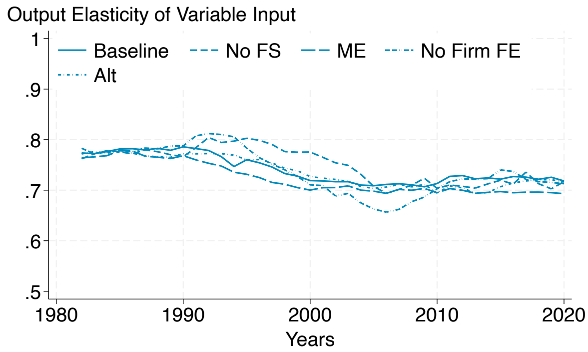
We plot the output elasticity of capital and labor for all these alternative estimation strategies in Figure C.3a and C.3b. The results are very similar regardless of the methodology applied. The output elasticity of capital has increased over time, in line with the findings of Chiavari and Goraya (2024). In contrast, output elasticity of variable input has declined in line with the findings of De Loecker et al. (2020).

Figure C.4a presents the evolution of the various markup measures for all the production function estimation techniques. In addition, we also apply the accounting profit approach, following Baqaee and Farhi (2020), where markups are calculated as the ratio of sales to total costs. Overall, we find that the elasticities of alternative production functions across inputs are broadly consistent, exhibiting similar trends despite some level differences. A similar conclusion holds for the different markup estimates, which display comparable trends over time, albeit with some differences in levels, as already noted in the literature (e.g., De Ridder et al., 2024).

Figure C.3: Alternative Production Functions



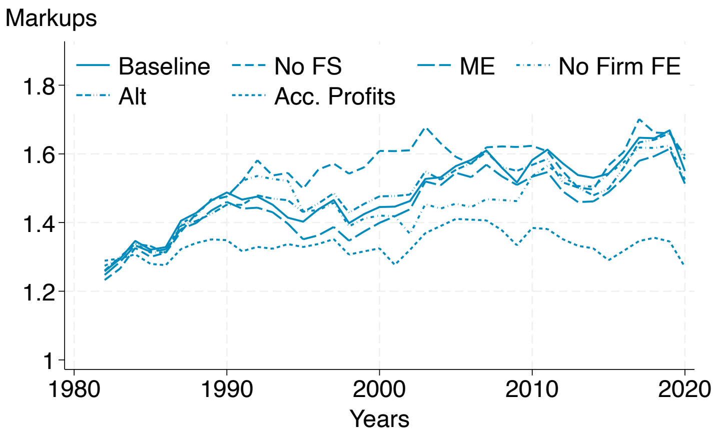
(a) Capital Elasticity



(b) Variable Cost Elasticity

Note. Figures C.3a, and C.3b show the evolution of elasticities for capital and variable costs across different production function estimation approaches.

Figure C.4: Alternative Markups Measures



(a) Markups

Note. Figure C.4a presents the evolution of the various alternative markup measures. Markups are presented as simple averages.

To further assess the similarity across different markup measures and move beyond simple mean comparisons, Tables C.2 and C.3 present the distribution of markups for the various alternative measures, along with their correlations. We find that the different markup measures exhibit surprisingly similar distributions across several moments and are highly and positively correlated. This is not entirely unexpected, as while there is some disagreement regarding the preferred method for estimating markups, there is broad consensus that most alternative measures display a positive trend and share comparable properties (e.g., Syverson, 2024). In conclusion, since the different measures exhibit very similar trends and only slight variations in their initial levels—differences that are not central in our methodology that focuses on trends relative to initial year–we conclude that neither the choice of markup measures nor the production function estimation methods significantly impact our results.

Table C.2: Summary Statistics for Alternative Markup Estimates

VARIABLES	(1) mean	(2) p10	(3) p50	(4) p90	(5) sd
Alter. Intangible Measure	1.521	0.829	1.199	2.413	1.134
No Firm FE	1.503	0.808	1.201	2.386	1.088
Baseline + ME	1.485	0.805	1.177	2.347	1.103
No First-Stage	1.561	0.817	1.223	2.478	1.248
Baseline	1.523	0.834	1.204	2.409	1.125
Accounting Approach	1.221	0.879	1.169	1.660	0.390

Table C.3: Correlations for Alternative Markup Estimates

	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	1.000	0.970	0.997	0.993	0.999	0.571
No First-Stage	0.970	1.000	0.978	0.958	0.971	0.565
Baseline + ME	0.997	0.978	1.000	0.989	0.997	0.570
No Firm FE	0.993	0.958	0.989	1.000	0.991	0.567
Alter. Intangible Measure	0.999	0.971	0.997	0.991	1.000	0.570
Accounting Approach	0.571	0.565	0.570	0.567	0.570	1.000

C.3 Additional Validations and Measurements of Risk Premium

Here, we present additional validations of our firm-level risk premium measure, demonstrating that it aligns well with standard theoretical predictions. We then show that alternative measures of firm-level risk premia exhibit a similar decline over time, supporting the robustness of our conclusions.

Validations. To validate our estimates of firm-level risk premia, we examine its correlation with firm-level equity returns, tangible and intangible capital used in production, and the ratio of tangible and intangible capital to variable costs. Theory predicts that firms with higher risk premia on capital should exhibit higher equity returns (David et al., 2022), lower levels of tangible and intangible capital due to a higher user cost of capital, and a lower ratio of tangible and intangible capital to variable costs, as risk premia affect the user cost of capital inputs but not of variable inputs.

Table C.4: Correlations Between Risk Premia and Other Firm-Level Variables

Dependent Variable	r^e	k^T	k^I	k^{all}	k^T/ℓ	k^I/ℓ	k^{all}/ℓ
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Risk Premium, ζ	0.149**	-0.180***	-0.292***	-0.346***	-1.516***	-1.403***	-1.569***
	(0.069)	(0.057)	(0.060)	(0.053)	(0.073)	(0.068)	(0.068)
Fixed Effect							
Firm	✓	✓	✓	✓	✓	✓	✓
Sector × Year	✓	✓	✓	✓	✓	✓	✓
Controls							
Age^2	✓	✓	✓	✓	✓	✓	✓
Observations	94,478	94,478	94,478	94,478	94,478	94,478	94,478

Note. All dependent variables but the equity return are in logs. Standard errors are clustered at the industry-year level and reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively.

Table C.4 presents the correlations between our firm-level risk premium on capital and firm-level equity returns, tangible and intangible capital used in production, as well as the ratio of tangible and intangible capital to variable costs. We find that all correlations are statistically significant, and all align with standard theoretical predictions.

To further validate our measure, we test an additional prediction highlighted by David et al. (2022). Sectors with more dispersed risk premia are expected to exhibit higher proxies of misallocation, given by greater dispersion in revenue-based marginal products of both types of capital and in revenue productivity, which is simply the geometric average of the two. This is the case because dispersion in risk premia implies dispersion in the user cost of both types of capital. Table C.5 tests and confirms these predictions in the data, showing that dispersion in risk premia is positively and statistically significantly associated with the dispersion of proxies for misallocation used in the literature. Overall, this evidence, along with the findings presented above, suggests that our measure captures many desirable properties of a good risk premium measure.

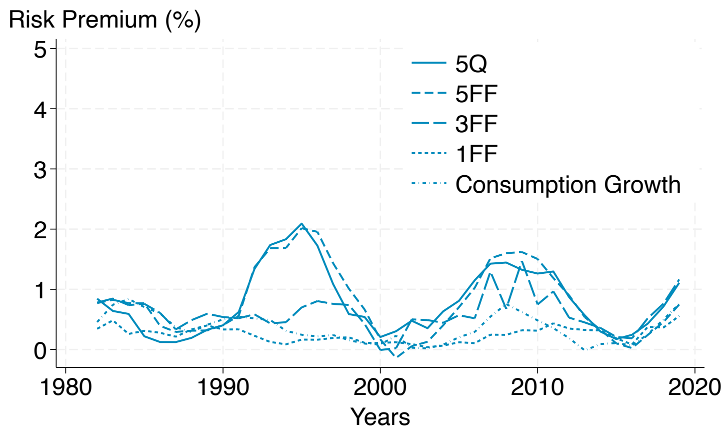
Robustness. Here, we present the evolution of the risk premium obtained using alter-

Table C.5: Risk Premia Dispersion and (Mis)Allocation

Dependent Variable	$\sigma(ARPK^{all})$ (1)	$\sigma(ARPK^{T})$ (2)	$\sigma(ARPK^{I})$ (3)	$\sigma(TFPR)$ (4)
Std. Risk Premium, $\sigma(\zeta)$	1.446***	1.452***	1.376***	0.075
	(0.180)	(0.179)	(0.164)	(0.054)
Fixed Effect				
Sector	✓	✓	✓	✓
Year	✓	✓	✓	✓
Observations	7,743	7,743	7,743	7,743

Note. Standard errors are clustered at the industry-year level and reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively.

Figure C.5: Alternative Risk Premium Measures



Note. Figure C.5 illustrates the evolution of the average capital-weighted risk premium across various factor models, including the baseline model, the Fama-French 5-factor model, the Fama-French 3-factor model, the Fama-French 1-factor model, and the consumption CAPM.

native factor models commonly found in the literature, including the 5-factor, 3-factor, and 1-factor models from Fama and French (2023), as well as the consumption CAPM. Figure C.5 illustrates the evolution of the risk premium derived from these different factor models. Overall, they exhibit a quantitatively similar trend in the average capital-weighted risk premium over time compared to the model proposed by Hou et al. (2015), suggesting that the specific asset pricing model employed does not significantly influence the trend of the risk premium.

Additionally, to further validate the robustness of our results, we demonstrate in Table C.6 that the various firm-level risk premia derived from different asset pricing models are positively and highly correlated. This suggests that, despite some differences, they all capture a similar source of underlying firm risk, beyond just their average evolutionary trends.

Table C.6: Correlation Matrix for Risk Premiums

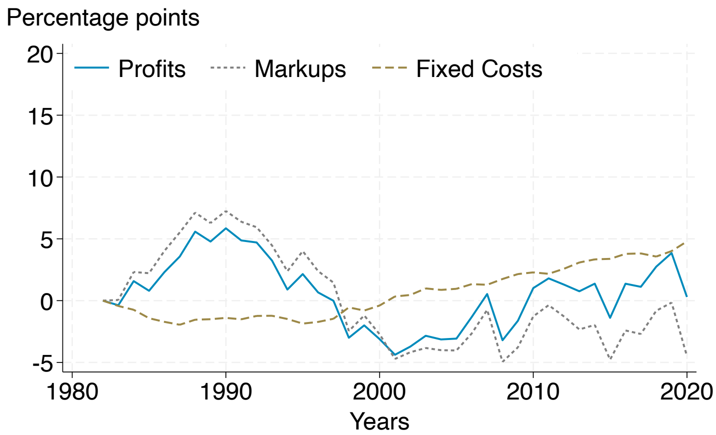
	MAIN	5FF	3FF	1FF	CCAPM
MAIN	1.00	0.75	0.63	0.37	0.54
5FF	0.75	1.00	0.66	0.38	0.57
2FF	0.63	0.66	1.00	0.51	0.59
1FF	0.37	0.38	0.51	1.00	0.59
CCAPM	0.54	0.57	0.59	0.59	1.00

D Robustness on the Micro-Anatomy of Return on Capital and Risk-Free Rate Divergence

D.1 Further Results on Main Decomposition

Here, we break down the evolution of profits to highlight the roles of markups and fixed costs. Figure D.6 illustrates this relationship, showing that profits' upward trend is both driven by the rise in markups and a decline in fixed costs.

Figure D.6: Profits, Markups, and Fixed Costs



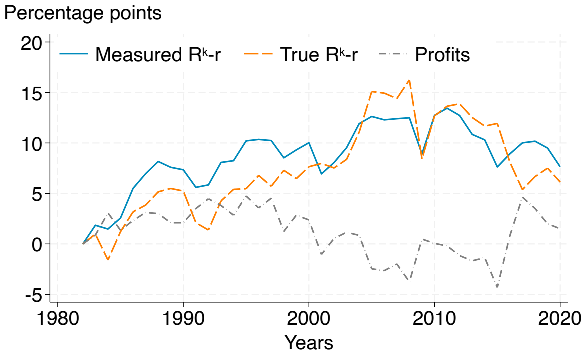
Note. Figure D.6 displays the progression of profits over time, breaking down the overall profit evolution into contributions from markups and fixed costs.

D.2 Robustness Exercises of Section 5.1

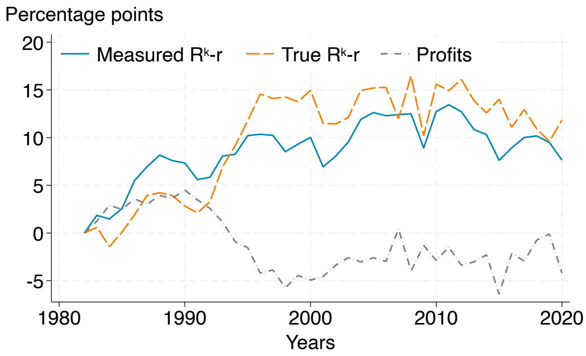
Figures D.7a, D.7b, D.7c and D.7c show the decomposition of the measured return on capital into the profit rate and the true return on capital for different robustness exercises. In Figure D.7a, we estimate the production function without the first stage. In Figure D.7b, we estimate a production function with measurement error. In Figure D.7c, we estimate a production function without firm fixed effects. In Figure D.7c, we measure the production function with an alternative measure of intangible capital as explained in section C.2. All figures show that true return on capital minus the risk-free rate was increasing until the late 2000s and then started declining over the next 10 years. Similarly, profit rates played a quantitatively less important role in the divergence between the measured return on capital and the risk-free rate. There was a sharp increase in profits until the late 1990s, and then it started declining sharply until 2010.

Figures D.8a, D.8b, D.8c and D.8d show the decomposition of the true return on capital into the risk premium, profits and the capital-specific wedge for different robustness exercises. In Figure D.8a, we estimate the production function without the first stage. In Figure D.8b, we estimate a production function with measurement error. In Figure D.8c, we estimate a production function without firm fixed effects. In Figure D.8d, we measure the production function with an alternative measure of intangible capital as explained in section C.2. Al figures show that capital wedges played a quantitatively dominant role in the divergence between the true return on capital and the risk-free rate, as in the main text. Finally, for brevity, we do not report additional robustness tests for the baseline approach with alternative risk premium estimates. Figure C.5 in Appendix C.3 illustrates the evolution of the capital-weighted risk premium across the various asset pricing models used in our robustness analysis. All models display either a declining or stable trajectory over time. Since these risk premiums serve as shifting factors for the capital wedge in the decomposition, they do not alter the main finding: the capital wedge remains the primary quantitative driver.

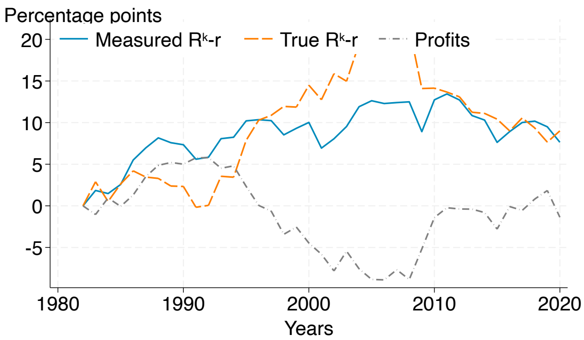
Figure D.7: Robustness Figure 7



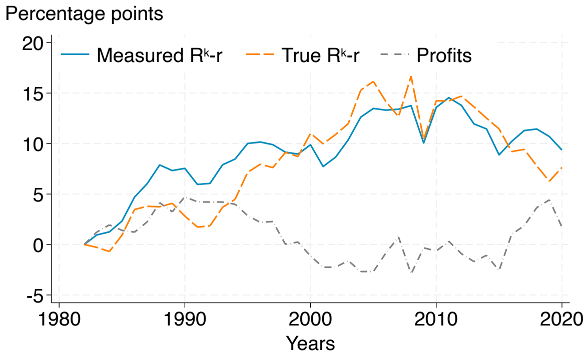
(a) Baseline without First Stage



(b) Measurement Error in Capital



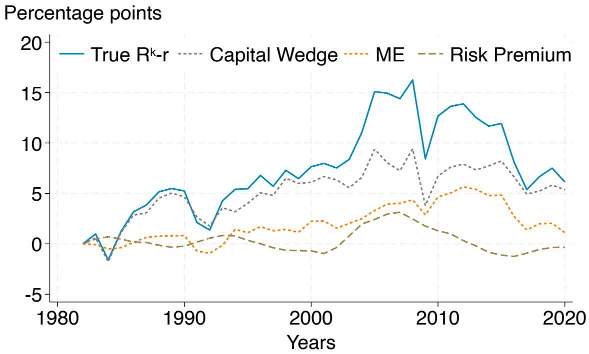
(c) Baseline without Firm FE



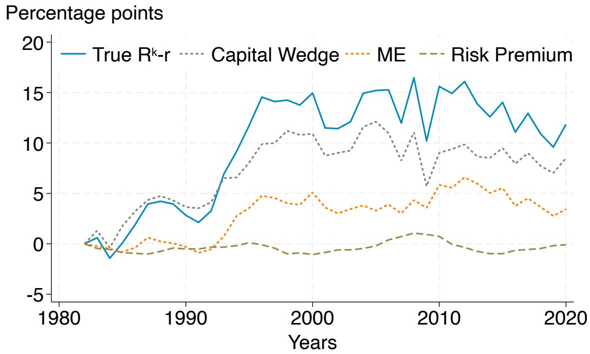
(d) Baseline with Alt. Intangible Measure

Note. Figures D.7a, D.7b, D.7c and D.7c show the decomposition of the measured return on capital into the profit rate and the true return on capital for different robustness exercises. In Figure D.7a, we estimate the production function without the first stage. In Figure D.7b, we estimate a production function with measurement error. In Figure D.7c, we estimate a production function without firm fixed effects. In Figure D.7c, we measure the production function with an alternative measure of intangible capital as explained in section C.2.

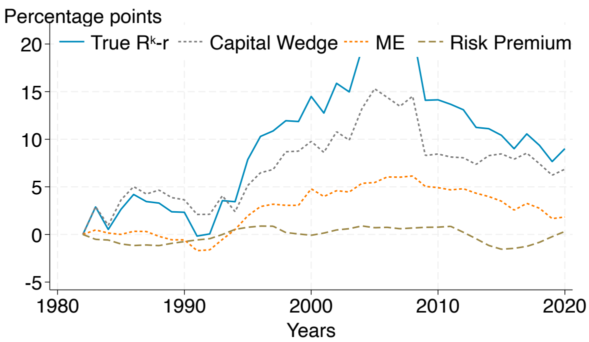
Figure D.8: Robustness Figure 8



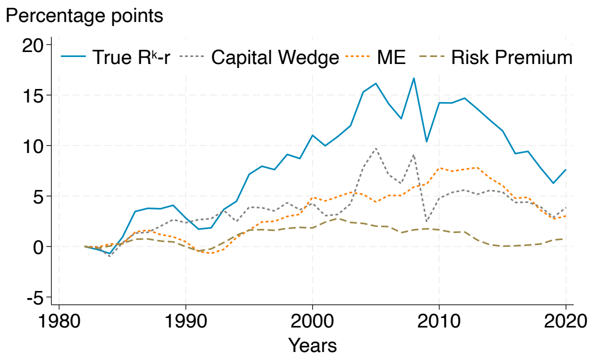
(a) Baseline without First Stage



(b) Measurement Error in Capital



(c) Baseline without Firm FE



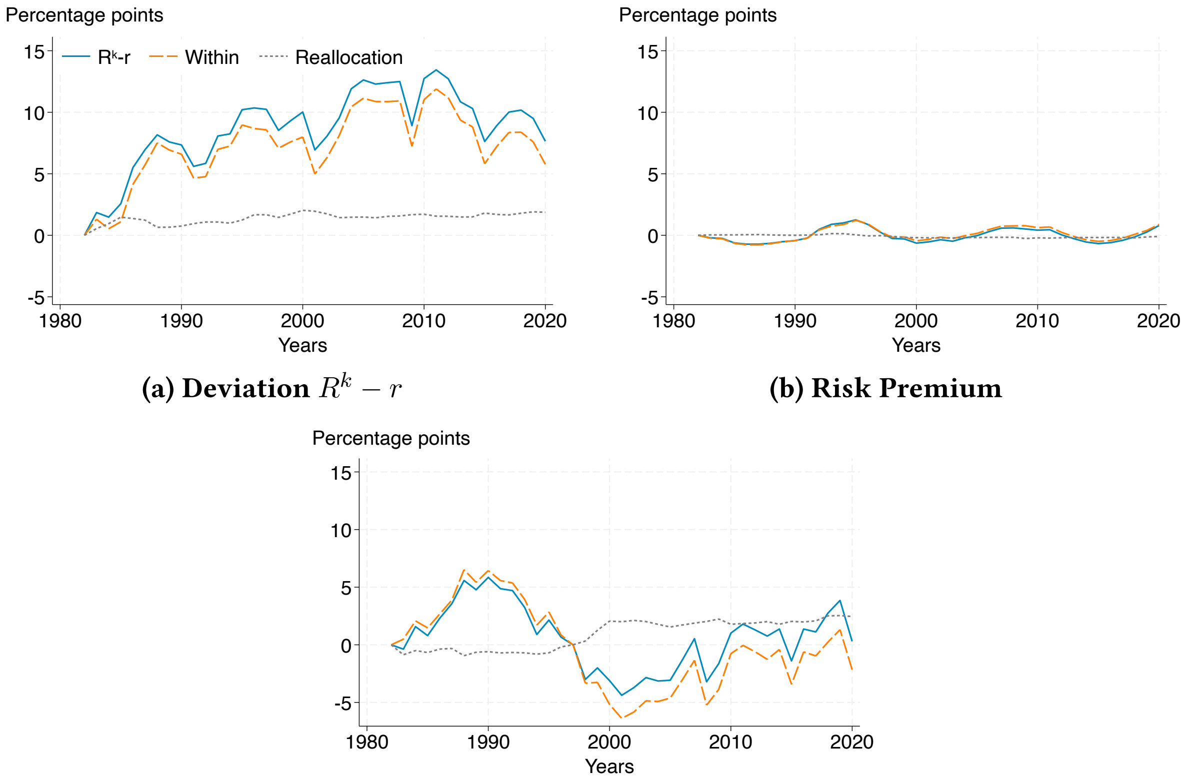
(d) Baseline with Alt. Intangible Measure

Note. Figures D.8a, D.8b, D.8c and D.8d show the decomposition of the true return on capital into the risk premium and the capital-specific wedge for different robustness exercises. In Figure D.8a, we estimate the production function without the first stage. In Figure D.8b, we estimate a production function with measurement error. In Figure D.8c, we estimate a production function without firm fixed effects. In Figure D.8d, we measure the production function with an alternative measure of intangible capital as explained in section C.2.

D.3 Additional Findings of Sectoral Decomposition

Figures D.9a, D.9b, and D.9c illustrate the sectoral decomposition of the Deviation $R^k - r$, risk premium, and profits. Overall, we find that the Within component of the sector-level decomposition is the primary driver of the evolution of these factors over time. This is consistent with the main text findings for the intangible capital wedge, which also highlights the main quantitative role of the Within component in driving overall changes.

Figure D.9: Tangible Wedge, Risk Premium, and Profits Sectoral Decomposition



(a) Deviation $R^k - r$

(b) Risk Premium

(c) Profits

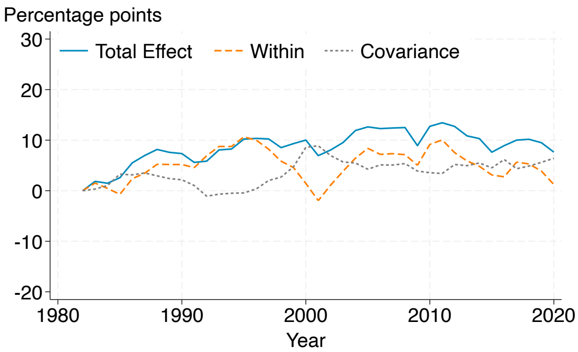
Note. Figures D.9a, D.9b, and D.9c show the sectoral decomposition of the over difference between the return on capital and risk-free rate, risk premium, markups, and fixed costs.

D.4 Additional Findings of Firm-Level Decomposition

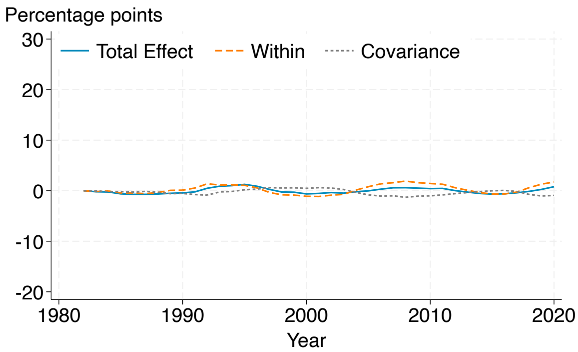
Firm-level decomposition of other variables. In the main text, we presented the firm-level decomposition of the intangible capital wedge. Here, we extend this analysis to provide a firm-level decomposition for other variables in our model: the tangible capital wedge, the risk premium, markups, and fixed costs.

Figures D.10a, D.10b, and D.10c present the firm-level decomposition of the Deviation $R^k - r$, risk premium, and profits, respectively. Several insights emerge. First, the overall difference between return on capital and risk-free rate $R^k - r$ is primarily driven by the within effect except for a few years. Second, the changes in the risk premium result from the Within effect. Third, the rise in profits is entirely attributable to the covariance component–the high markup or more profitable firms are becoming more important in the economy. This finding aligns with existing literature on the rise of markups.

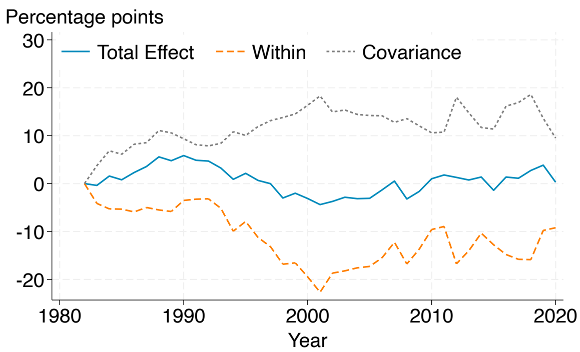
Figure D.10: Tangible Wedge, Risk Premium, Markups, and Fixed Costs Firm-Level Decomposition



(a) Deviation $R^k - r$



(b) Risk Premium



(c) Profits

Note. Figures D.10a, D.10b, and D.10c show the firm-level decomposition of the Deviation $R^k - r$, risk premium, and profits.

Comparison with the literature. In the main text, we show that different cohorts exhibit varying levels of intangible intensity, especially at younger ages. Here, we provide the regression results for the section **??** in Table D.7 and D.8. In Table **??**, we define a cohort of firms that are within 5 years, and birth is defined by the first year of appearance in the Compustat data. In Columns 1 and 2, we provide the coefficient for the baseline regression whose coefficients are plotted in the Figure 11a and 11b. In Columns 3 and 4, we provide the coefficient from the regression where we exclude the young firms to avoid the dependence of the very young firms, whose intangible capital is sensitive to the assumption on the initial stock of capital that is used for the perpetual inventory method.

We repeat the exercise in Table D.8, but we define firm birth by their IPO year. Here, the number of observations is much smaller because IPO year is not defined for all firms. However, our results remain qualitatively similar.

Table D.7: Cohort effects for Frictions and Intangibles

	(1) Capital Frictions	(2) Intangibles	(3) Capital Frictions	(4) Intangibles
1980 cohort	0.044**	0.053***	0.023***	0.039***
	(0.010)	(0.005)	(0.004)	(0.002)
1990 cohort	0.088**	0.092***	0.044**	0.060***
	(0.020)	(0.012)	(0.012)	(0.005)
2000 cohort	0.129**	0.123***	0.047*	0.078***
	(0.029)	(0.019)	(0.017)	(0.009)
2010 cohort	0.206***	0.197***	0.045*	0.122***
	(0.032)	(0.020)	(0.018)	(0.010)
Observations	157,177	153,757	135,817	135,817
R-squared	0.072	0.297	0.188	0.273
R-squared	0.092	0.293	0.189	0.272
Sector FE	yes	yes	yes	yes
Year FE	yes	yes	yes	yes
Age>3	No	No	yes	yes

Note. In Table D.7, we define a cohort of firms that are within 5 years. "Capital Frictions" is defined as adjusted capital wedge τ^{adjusted}. "Intangibles" is defined as the intangible investment share of capital. Birth is defined by the first year of appearance in the Compustat data. In Columns 1 and 2, we provide the coefficient for the baseline regression whose coefficients are plotted in the Figure 11a and 11b. In Columns 3 and 4, we provide the coefficient from the regression where we exclude the young firms. Standard errors are reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively.

Table D.8: Cohort effects for Frictions and Intangibles: Robustness

	(1) Capital Frictions	(2) Intangibles	(3) Capital Frictions	(4) Intangibles
1980 cohort	0.016	0.033***	0.011	0.029***
	(0.009)	(0.005)	(0.009)	(0.004)
1990 cohort	0.074***	0.074***	0.054***	0.050***
	(0.009)	(0.005)	(0.008)	(0.004)
2000 cohort	0.072***	0.106***	0.045***	0.075***
	(0.009)	(0.007)	(0.008)	(0.005)
2010 cohort	0.071***	0.168***	0.043**	0.125***
	(0.013)	(0.008)	(0.011)	(0.006)
Observations	67,767	67,566	60,160	60,160
R-squared	0.208	0.313	0.253	0.297
Sector FE	yes	yes	yes	yes
Year FE	yes	yes	yes	yes
Age>3	No	No	yes	yes

Note. In Table D.8, we define a cohort of firms that are within 5 years. "Capital Frictions" is defined as adjusted capital wedge τ^{adjusted}. "Intangibles" is defined as the intangible investment share of capital. In Columns 1 and 2, we provide the coefficient for the baseline regression. In Columns 3 and 4, we provide the coefficient from the regression where we exclude the young firms. Birth is defined by the IPO year. Standard errors are reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively.